Techniques for Solving Math Word Problems

Presenter

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Objective for the Session

- Learn about and experience techniques for improving students' access to and success with math word problem.
  - Seven categories of techniques
  - 20+ techniques

Seven Categories of Techniques

I. General Ideas

II. Understand but Don't Yet Solve

III. Using Visuals

IV. Getting Student Buy-in

V. Adapting Problems

VI. Identifying Problem Types

VII. Monitor, Process and Practice

CCSSM Practice 4 – Model with Mathematics

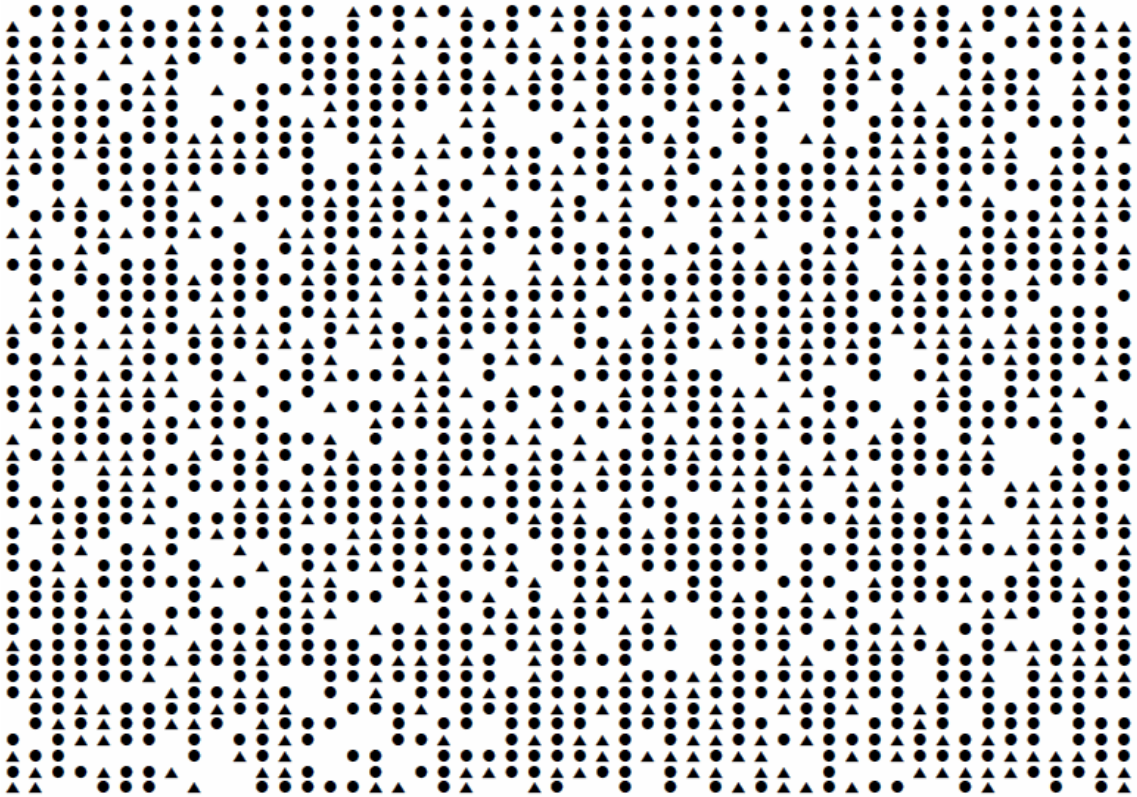
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

- In early grades, this might be as simple as writing an addition equation to describe a situation.
- In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
- By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

MARS Task: Counting Trees

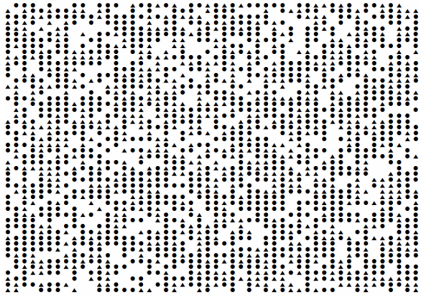
www.map.mathshell.org/tasks

- This diagram shows some trees in a tree farm.
- Circles ● show old trees and triangles ▲ show young trees.



Counting Trees

How many trees are there of each type?



- **Pre-Work:**
  - **E**stimate/guess the number of old trees and number of new trees. (30 seconds – WRITE IT DOWN)
  - **S**hare guess and reasoning with a partner.

- **Solve:**
  1. **D**iscuss plan for how to make a good estimate with a partner.
  2. **S**olve without counting every tree (3-4 minutes).

- **Post-Work:**
  - **C**ompare solution and methods with other partners.
  - **S**tronger: share one idea about how they could make explanation stronger.

Handout p. 1

Techniques Used with *Counting Trees*

E S – D S – C S

- **E**stimate/guess – **S**hare your reasoning
  - Estimate or guesstimate an answer, write this down, then share with someone else and explain your reasoning.
- **D**iscuss and plan – **S**olve
  - Discuss the problem with a partner and plan how to solve it, then put your plan into action and solve the problem.
- **C**ompare– Make **S**tronger
  - Compare your answer and process with another pair of students and make one suggestion for how their explanation or process could be improved.

George Polya or UPS Check

- **U**nderstand the problem
- **P**lan how to solve the problem
- **S**olve the problem
- **C**heck or look-back to answer and sensibleness of your answer

Assessing Counting Trees

- Rubrics – apply sometimes
- Giving students credit for evidence of understanding
- Rubrics such as this identify key benchmarks in solving the problem and assign credit.
- Easy to apply once created
- Be flexible – adapt after you see student work

Handout p. 2

Density of Text

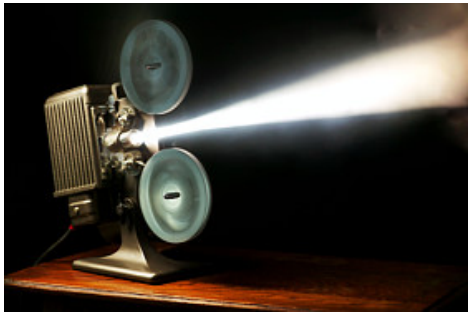
Mathematics is the most difficult content area material to read because there are more concepts per word, per sentence, and per paragraph than in any other subject; the mixture of words, numerals, letters, symbols, and graphics requires the reader to shift from one type of vocabulary to another.

—Braselton & Decker, 1994

Amplify not Simplify

Teachers can foster students' sense-making by amplifying rather than simplifying, or watering down, their own use of disciplinary language.

Zwiers et al. 2017



Example:

Simplifying: Continually referring to the numerator as the "top number."

Amplifying: Build on a student saying "the top number" by asking "what do we call that?", refer to an anchor chart, and/or have the class chorally repeat the term "numerator" while visually putting their hands above their heads.

Focus on the Math

Research shows that ELs, even as they are learning English, can participate in discussions where they grapple with important mathematical content. Instruction for this population should not emphasize low-level language skills over opportunities to actively communicate about mathematical ideas.

Moschkovich, 2012

IES Sample of Steps for Problem Solving

1. Identify the givens and goals of the problem.
2. Identify the problem type.
3. Recall similar problems to help solve the current problem.
4. Use a visual to represent and solve the problem.
5. Solve the problem.
6. Check the solution.

Institute of Education Sciences (IES), *Improving Mathematical Problem Solving in Grades 4–8*, 2012

Problem Solving

Problem solving involves reasoning and analysis, argument construction, and the development of innovative strategies. These abilities are used not only in advanced mathematics topics—such as algebra, geometry, and calculus—but also throughout the entire mathematics curriculum beginning in kindergarten.

Institute of Education Sciences (IES), *Improving Mathematical Problem Solving in Grades 4–8*, 2012

I. General Techniques

1. Polya technique or *UPS Check*
2. Estimate-Share / Discuss Plan – Solve / Compare – make Stronger
3. Dissecting problems (highlight and identify key information)
4. Teach and practice a routine (Kady Dupre)

   at Laura Candler's Teaching Resources

   https://www.lauracandler.com/math-word-problems/

Teach a Problem-Solving Routine

Kady Dupre

My favorites always include a "before, during, and after" mindset

- Use it often (daily if possible)
- Incorporate "Turn & Teach" (Students orally explain their thinking and process to a partner)
- Allow for "Strategy Share" after solving (Selected students explain their method and thinking)

www.lauracandler.com/math-word-problems/

II. Understand but Don't Yet Solve

5. Last sentence is the goal
6. Three Reads
7. Missing Number(s)
8. Parallel Problems
9. Represent visually but don't solve
10. Compare problems side by side (Kady Dupre)

    at Laura Candler's Teaching Resources
    https://www.lauracandler.com/math-word-problems/

The Last Sentence is the Goal

Read or emphasize the last sentence first.

A rectangular container that has a length of 30 cm, a width of 20 cm, and a height of 24 cm is filled with water to a depth of 15 cm. When an additional 6.5 liters of water are poured into the container, some water overflows. **How many liters of water overflow the container?**

Use words, pictures, and numbers to explain your answer. (Remember: 1 cm^3 = 1 mL.)

EngageNY/Eureka Math Grade 5 Module 5 End of Module Assessment #3

Three Reads (Mathematical Language Routine #6) (Zwiers et al., 2017)

1) Students read the situation with the goal of comprehending the text (describe the situation without using numbers). ***[Get the gist]***

2) Students read the situation with the goal of analyzing the language used to present the mathematical structure. ***[Understand language]***

3) Students read the situation in order to brainstorm possible mathematical solution methods. ***[Plan]***

Understanding Language/Stanford Center for Assessment, Learning and Equity at Stanford University,. 2017

Missing Number – Examples

Pocket Money

Marcos has $17 altogether. He has $ ____ in his hand and the rest of the money in his pocket. How much money does he have in his pocket?

Cell Phone Deal

A phone originally sells for $ ____. It is now on sale for 1/5 off the original price. April has a coupon for an extra 10% off the sale price. To the nearest dollar, how much less than the original price will April pay for the phone?

from Ready Math Grade 7 Lesson 16

Parallel Problems

Cell Phone Deal

- What is the problem asking for? How might you solve the problem? No credit for solving this version.

A phone originally sells for $500. It is now on sale for 1/5 off the original price. April has a coupon for an extra 10% off the sale price. To the nearest dollar, how much less than the original price will April pay for the phone?

Adapted from Ready Math Grade 7 Lesson 16

Solve this version of the problem

A phone originally sells for $245. It is now on sale for 1/5 off the original price. April has a coupon for an extra 10% off the sale price. To the nearest dollar, how much less than the original price will April pay for the phone?

from Ready Math Grade 7 Lesson 16

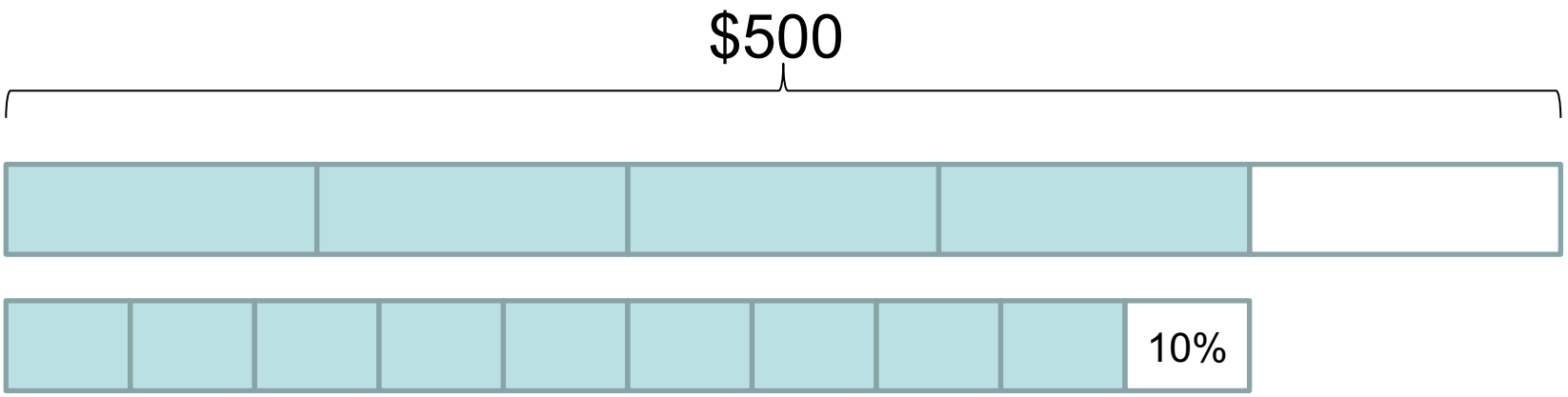
Represent Visually but Don't Solve

Cell Phone Deal

Create a tape diagram representing the problem situation.

A phone originally sells for $500. It is now on sale for 1/5 off the original price. April has a coupon for an extra 10% off the sale price. To the nearest dollar, how much less than the original price will April pay for the phone?

from Ready Math Grade 7 Lesson 16



Compare Problems Side by Side
Kady Dupre (Monkeys #2.1)

Monkey Joe picked 7 bananas in the morning and 19 bananas in the afternoon. How many bananas did he pick?	Monkey Joe picked 7 bananas in the morning and 19 bananas in the afternoon. How many more bananas did he pick in the afternoon?
Sketch it.	Sketch it.
Solve it.	Solve it.

How are the problems different?

www.lauracandler.com/math-word-problems/

III. Using Visuals

11. Diagramming, creating visual models
    - *Tape diagrams*
    - *Area models*
    - *Number lines*
    - *"Read/Draw/Write" (EngageNY/Eureka Math)*
12. Matching models to problems
13. Matching expressions with terminology

Represent Visually and Solve

Cell Phone Deal

Create a tape diagram representing the problem situation.

A phone originally sells for $245. It is now on sale for 1/5 off the original price. April has a coupon for an extra 10% off the sale price. To the nearest dollar, how much less than the original price will April pay for the phone?

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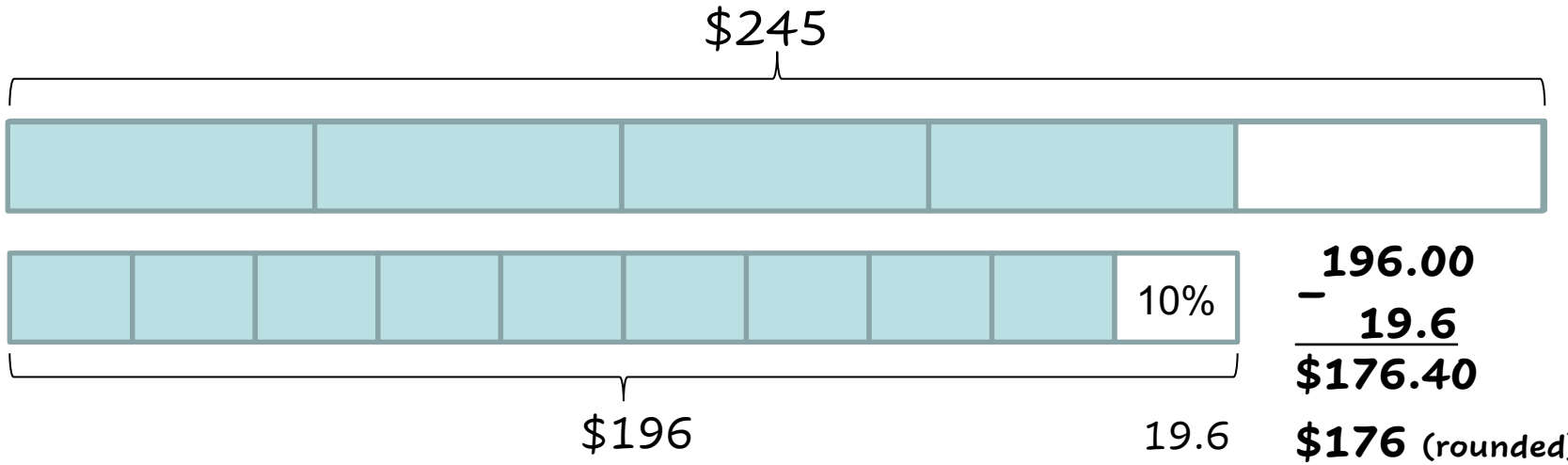


196.00
− 19.6
$176.40

$176 (rounded)

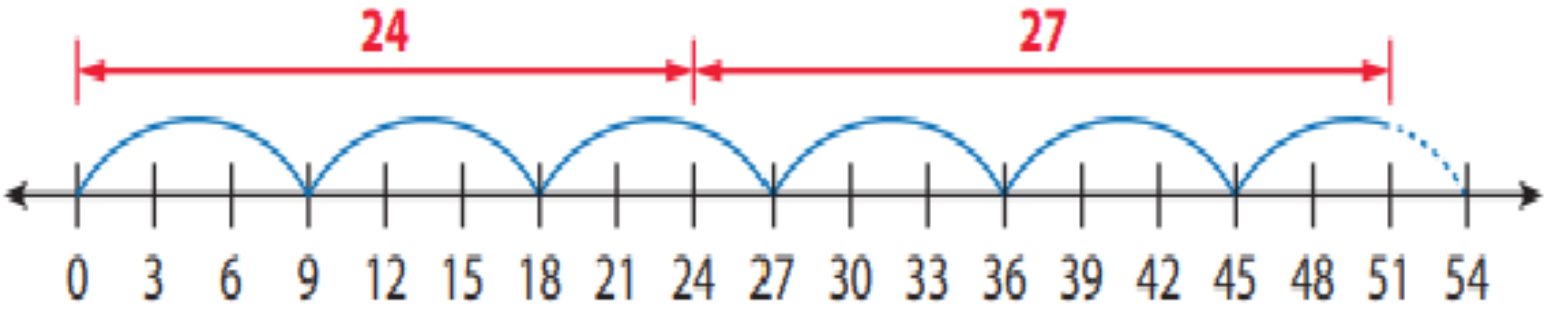
Read Draw Write (RDW)

When faced with story problems young children will often arbitrarily add whatever numbers they see. The RDW process short-circuits this impulse by giving students tools to think about and model the relationships presented in a given situation. This enables them to solve problems accurately.

Lisa Watts-Lawton in Eureka Math blog at greatminds.org/math/blog/eureka/post/problem-solving-the-rdw-way

Visually with a Number Line

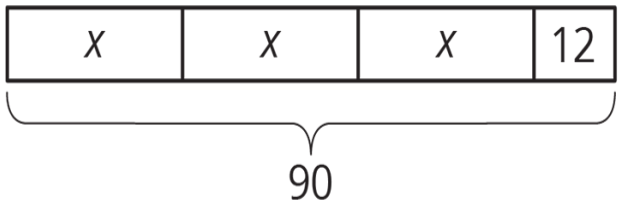
Ms. Dennison is packing up the books in her classroom for the summer. Each box holds 9 books. She has 24 math books and 27 science books to pack. How many boxes will she need?



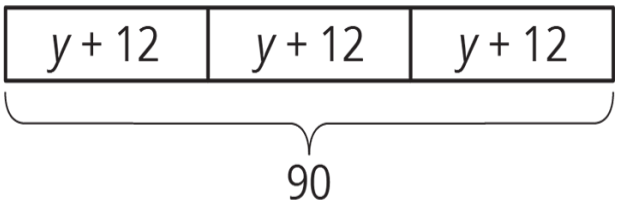
Ready Classroom Grade 4 Lesson 10 p. 94 (2020)

Matching Models to Problems

A



B



B **Story 1:** Lin had 90 flyers to hang up around the school. She gave 12 flyers to each of three volunteers. Then she took the remaining flyers and divided them up equally between the three volunteers.

A **Story 2:** Lin had 90 flyers to hang up around the school. After giving the same number of flyers to each of three volunteers, she had 12 left to hang up by herself.

1. Which diagram goes with which story? Be prepared to explain your reasoning.

Open Up Resources (OUR) Grade 7 Unit 6 Lesson 6 Activity 3

Match the word problem with the correct expressions at the bottom.

1. Write an expression to show a number increased by $\mathbf{11}$. $y + 11$
2. Write an expression to show a number decreased by $\mathbf{11}$. $y - 11$
3. Write an expression to show y less than $\mathbf{3.5}$. $3.5 - y$
4. Write an expression to show the sum of x and y reduced by $\mathbf{11}$. $(x + y) - 11$
5. Write an expression to show $\mathbf{5}$ less than y, plus x. $y - 5 + x$

$(x + y) - 11$	$y - 11$	$5 - y + x$	$3.5 - y$
$11 - (x + y)$	$y + 11$	$y - 5 + x$	$y - 3.5$

Adapted from EngageNY/Eureka Math Grade 6 Module 4 Lesson 9 (2016)

Matching Problems to an Expression

4 Which situation could be represented by the following expression? Circle all that apply.

$48 + 2x$

✓ **A** Sara's phone contract costs her $48 per month, but she pays an additional $2 for every minute she goes over her allotted minutes.

✓ **B** A fast food restaurant expects to use 48 eggs per day plus an additional 2 eggs for every customer coming in for breakfast.

C A florist began the day with 48 roses and sold approximately 2 roses per hour.

✓ **D** Visitors to an amusement park pay an entrance fee of $48 plus $2 for each ticket purchased for the rides.

Ready Classroom Grade 6 Lesson 19 Practice p. 201 (2020)

Handout p. 4

IV. Getting Student Buy-In

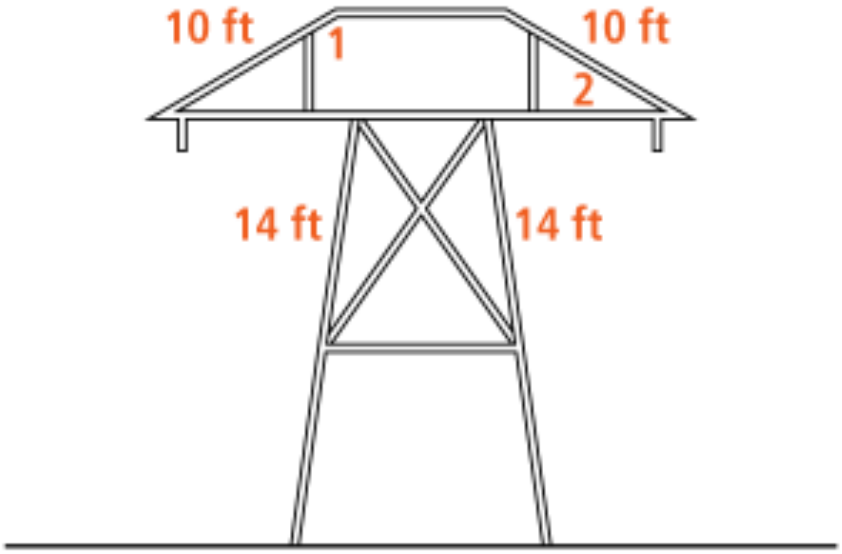
(9) Estimate/guess first and explain reasoning

14. Create word problems
15. Act it out
16. Vocabulary scavenger hunt
17. Could this be the answer? Why or why not?

Could This Be the Answer?

All horizontal beams of the high-voltage transmission tower are parallel to the ground. The top section is an isosceles trapezoid. The center section is an isosceles trapezoid.

If the measure of angle 1 = 128° what is the measure of angle 2?



Could 120° be the answer?
Why or why not?

Could 10° be the answer?
Why or why not?

Could 40° be the answer?
Why or why not?

Adapted from enVision Math Geometry Topic 6 Lesson 2 (2019)

V. Adapting Problems

18. Differentiate – different numbers
19. Bulleted version
20. Sentence frames
21. True/False and multiple choice with word problems

Differentiate (Kady Dupre) – Different Numbers (with Three Reads and Missing Numbers)

Board the Roller Coaster

Jackson needs to be 1 ¾ inches taller in order to ride the roller coaster. Since he can't wait, he puts on a pair of boots that adds 1 1/6 inches to his height and slips an insole inside the boot that adds half as much as the boot does to his height. Will this make Jackson appear tall enough to ride the roller coaster?

Adapted from EngageNY/Eureka Math Grade 5 Module 3 (2016)

1. Get the Gist | **2. Language?** | **3. Plan/solve**

Bulleted version

Board the Roller Coaster

Jackson needs to be 1 ¾ inches taller in order to ride the roller coaster.

- Since he can't wait, he puts on a pair of boots that adds 1 1/6 inches to his height, and
- slips an insole inside the boot that adds half as much as the boot does to his height.

Will this make Jackson appear tall enough to ride the roller coaster?

Adapted from EngageNY/Eureka Math Grade 5 Module 3 (2016)

Best Deal

One store is having a 50% off sale. Another store has a 40% discount, with an additional 15% off of the sale price. Which sale should you take advantage of if you want the best reduction on a sweater that costs $68.79?

Commit to ONE of the following without actually solving:

A. The first store will give me the best price on the sweater.

B. The second store will give me the best price on the sweater.

C. It doesn't matter. The cost of the sweater will be the same in both stores.

Handout p. 6

Turn and Talk with Sentence Frames

Partner share. Complete the following:

- *"What I first noticed about the problem was _____."*
- *"I believe my answer makes sense because _____."*

Best Deal – Part II

One store is having a 50% off sale. Another store has a 40% discount, with an additional 15% off of the sale price. Which sale should you take advantage of if you want the best reduction on a sweater that costs $68.79?

- **Individually:** Solve the problem. Show your work.
- **Pair-share:**
  - *I think the 50% discount is _____ than the 40% plus an added 15% discount off of the sale price because _____.*
  - Turn to a partner and share your statement.

Sentence Stems and Frames

- Specific sentence frames:
  A. The store with the best reduction is __ because ___.
  B. The sales at the two stores are similar because they both _____.
  C. The sales at the two stores are different because ___.
  D. Comparing the sales at the two stores is tricky because _____.

True False & Multiple Choice

3 Caroline charges \$15 per hour babysitting. Let h represent the number of hours she babysits and E represent how much she earns. Choose *True* or *False* for each statement.

a. $h + 15 = E$ is the equation that represents how much Caroline earns after h hours. ☐ True ☑ False

b. If Caroline babysits for 5 hours, she earns \$20. ☐ True ☑ False

c. $15h = E$ is the equation that represents how much Caroline earns after h hours. ☑ True ☐ False

d. If Caroline earned \$52.50, then she babysat for $3\frac{1}{2}$ hours. ☑ True ☐ False

e. $75f$ represents how much Caroline makes after f days babysitting 5 hours a day. ☑ True ☐ False

Ready Classroom Grade 6 Lesson 19 Practice p. 200 (2020)

VI. Identifying Problem Types

22. Identifying problem types

. . . Schoenfeld in his 1992 review of the literature concluded that attempts to teach students to use general problem-solving strategies (e.g., draw a picture, identify the givens and goals, consider a similar problem) generally had not been successful. He recommended that better results might be obtained by developing and teaching more specific problem-solving strategies (that link more clearly to classes of problems) . . .

Lesh & Zawojewski 2007

Addition & Subtraction Problem Types

	Result Unknown	Change Unknown	Start Unknown
Add To	2 + 3 = ?	2 + ? = 5	? + 3 = 5
Take From	5 – 2 = ?	5 – ? = 3	? – 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown
Put Together/ Take Apart	3 + 2 = ?	3 + ? = 5, 5 – 3 = ?	5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	2 + ? = 5, 5 – 2 = ?	2 + 3 = ?, 3 + 2 = ?	5 – 3 = ?, ? + 3 = 5

CCSSM 2010

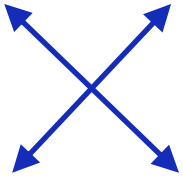
Multiplication & Division Problem Types

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
Equal Groups	3 x 6 = ?	3 x ? = 18 18 / 3 = ?	? x 6 = 18 18 / 6 = ?
Arrays, Area	3 x 6 = ?	3 x ? = 18 18 / 3 = ?	? x 6 = 18 18 / 6 = ?
Compare	3 x 6 = ?	3 x ? = 18 18 / 3 = ?	? x 6 = 18 18 / 6 = ?
General	$a \times b = ?$	$a \times ? = p, \ p / a = ?$	$? \times b = p, \ p / b = ?$

CCSSM 2010

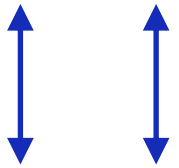
Schemas for Solving Proportions

$$\frac{12}{11} = \frac{40}{x}$$



$$\frac{4}{11} = \frac{12}{x}$$

$$\frac{4}{12} = \frac{5}{x}$$



Research shows that students improve on solving proportions when they explicitly learn about different solution schemes as shown above. However, this is only part of the bigger idea about proportionality—students must still recognize proportional situations and how quantities are related in a proportion.

Jitendra 2010

Types, Characteristics, and Representations of Functions

Types of Functions	Characteristics of Functions	Representations of Functions
• Linear • Quadratic • Exponential • Trigonometric	• Rate of change • Shape of graph • *y*-intercept • *x*-intercept • Maximums, minimums • Points of inflection • *Etc.*	• Data table • Graph • Equation • Verbal description • Context

VII. Monitor, Process and Practice

23. Monitor and respond as needed
24. Select and purposely share
25. Practice and/or extend

CORE MATH

Thank you!

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