Mathematical Fluency and Number Sense: Techniques, Access and Sustainability for All Students

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Essential Questions to Answer Today

• What are number sense and fluency, how are they related and why are they important?

• What activities build fluency and number sense together?

• What are important look-fors in the classroom?

• What role does developing fluency and number sense play with intervention and special education?
Fluency – There is always your fingers

Kid Snippets – "Math Class"

https://www.youtube.com/watch?v=KdxEAt91D7k
Kid Snippets – "Math Class"
What Is Fluency?

. . . procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) . . .

CCSSM, p. 6
Number Sense

• Fluidity and flexibility with numbers
• Sense of what numbers mean
• Ability to perform mental mathematics and to look at the world and make comparisons

Gersten & Chard 2001
Phone Number Equation

Using your phone number, without the area code, create at least one equation

- Use any combination of operations
- Create an expression with the first three numbers equal to an expression created with the last four numbers.

Example: 328-6537 → 8 x (3 + 2) = (6 x 5) + (3 + 7)

Example: 328-6537 → (3 – 2) x 8 = (7 – 6) x (5 + 3)
NAEP 4th Grade: Closest to $\frac{1}{2}$

Which fraction has a value closest to $\frac{1}{2}$?

A. $\frac{5}{8}$  
B. $\frac{1}{6}$  
C. $\frac{2}{2}$  
D. $\frac{1}{5}$

Which is the correct answer?
Which do you think is the most popular incorrect answer and why?

25% answered correctly (A)

40% chose C
Why Worry About Fluency?

• Real world
• Standards
• Assessments
• Impact on learning
Five Proficiency Strands in Mathematics

- Conceptual understanding
- Strategic competence
- Adaptive reasoning
- Productive disposition
- Procedural fluency

National Research Council 2001
# Fluency Chart for K–8

<table>
<thead>
<tr>
<th>Grade</th>
<th>Required Fluency</th>
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<tbody>
<tr>
<td>K</td>
<td>Add and subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>Add and subtract within 10</td>
</tr>
</tbody>
</table>
| 2     | Add and subtract within 20 (mentally)  
          Add and subtract within 100 |
| 3     | Multiply and divide within 100  
          Add and subtract within 1,000 |
| 4     | Add and subtract multidigit whole numbers using standard algorithms |
| 5     | Multiply multidigit whole numbers using standard algorithm |
| 6     | Add, subtract, multiply, and divide multidigit numbers (including decimals)  
          using standard algorithms |
| 6-8   | Compute with positive and negative fractions and decimals |
Assessments and Fluency

Assessments typically include three key areas:

- Facts and procedures
- Concepts
- Applications
### How Many Number Facts? (Handout p. 5)

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<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
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</table>
## Limits of Working Memory

### Capacity and Time

<table>
<thead>
<tr>
<th>Age (Years)</th>
<th>Average Capacity Range (Chunks of Information)</th>
<th>Average Time Limit for Retention</th>
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<tbody>
<tr>
<td>Younger than 5</td>
<td>2 + 1</td>
<td>Unknown</td>
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<tr>
<td>From 5 to 14</td>
<td>5 + 2</td>
<td>5 to 10 minutes</td>
</tr>
<tr>
<td>Older than 14</td>
<td>7 + 2</td>
<td>10 to 20 minutes</td>
</tr>
</tbody>
</table>


$$7(x + 8) = 7x + 56$$
Making Room in Working Memory

Our ability to think would be limited indeed if there were not ways to overcome the space constraint of working memory. One of the more important mechanisms is the development of automaticity. When cognitive processes... become automatic, they demand very little space in working memory, they occur rapidly, and they often occur without conscious effort.

Daniel Willingham, American Educator, Spring 2004
Recommendations from Research

• *Provide 10 minutes of daily practice to strengthen needed fluency with facts and procedures.*
  
  Gersten et al., (2009)

• *Distributed or spaced practice, repeated practice of previously learned knowledge over “a long period of time,” has a high effect size of 0.71.*
  

• *Fluency with whole numbers and fractions are part of a critical foundation for learning algebra.*
  
Rote and Elaborate Rehearsal

• **Rote rehearsal:** Memorization without continuing to think through an idea or fact.

• **Elaborate rehearsal:** Making sense of ideas and information. The learner processes and reprocesses information to connect it together, to connect it to prior learning, and to assign meaning to it. Elaborate rehearsal is necessary for students to probe the deeper meaning and interrelationships of mathematical concepts.
Why We Need Elaborate Rehearsal

Memories are formed as the residue of thought. You remember what you think about, but not every fleeting thought—only those matters to which you really devote some attention.

Daniel Willingham 2008
Fluency Activities with Number Sense in Mind
Making 24

Use the four numbers and any combination of math operations to get a result of 24.

Use 1, 1, 4 and 6 to make 24:  \[ 1 \times 1 \times 4 \times 6 = 24 \]

Use 8, 8, 9, and 11 to make 24:  \[ 9 / (11-8) \times 8 = 24 \]

You try it: use 6, 6, 11, and 13 to make 24:  \[ 13 \times (6 / 6) + 11 = 24 \]

Great website: https://www.4nums.com/
Card Games

Examples

– Integer Addition War
– Hit the Target
– Addition War
– Fraction War
Hit the Target

Target is 12

\[
\begin{align*}
5 + 6 + 1 & \quad 2 \times 6 & \quad 2(5 + 1) \\
(8 - 6) \times 5 \times 1 + 2
\end{align*}
\]
KenKenPuzzle.com
Individual White Board Review

1. \( \frac{1}{2} = \frac{2}{4} \)

2. \( \frac{1}{2} = \frac{5}{10} \)

3. \( \frac{1}{4} = \frac{2}{8} \)

4. \( \frac{1}{4} = \frac{5}{20} \)

5. \( \frac{3}{4} = \frac{15}{20} \)

6. \( \frac{1}{4} = \frac{3}{12} \)

7. \( \frac{3}{9} = \frac{1}{3} \)

8. \( \frac{4}{12} = \frac{2}{6} \)
**SPRINTS**

It’s not a race.

It’s not a test.

It’s practice.

It’s learning strategies and building fluency.

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### A

**Write the Missing Factor**

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<td>1.</td>
<td>10 = 5 × ____</td>
<td>21.</td>
<td>28 = 7 × ____</td>
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<tr>
<td>2.</td>
<td>10 = 2 × ____</td>
<td>22.</td>
<td>23 = 2 × 2 × ____</td>
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<td>3.</td>
<td>8 = 4 × ____</td>
<td>23.</td>
<td>23 = 2 × ____ × 2</td>
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<td>4.</td>
<td>9 = 3 × ____</td>
<td>24.</td>
<td>23 = ____ × 1 × 2</td>
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<td>5.</td>
<td>6 = 2 × ____</td>
<td>25.</td>
<td>36 = 3 × 3 × ____</td>
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<td>6.</td>
<td>6 = 3 × ____</td>
<td>26.</td>
<td>9 = 3 × 3 × ____</td>
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<tr>
<td>7.</td>
<td>12 = 6 × ____</td>
<td>27.</td>
<td>9 = 4 × 3 × 3 × ____</td>
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<td>8.</td>
<td>12 = 3 × ____</td>
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<td>9 = 4 × 3 × ____</td>
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<td>9.</td>
<td>12 = 4 × ____</td>
<td>29.</td>
<td>9 = 4 × 3 × ____</td>
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<td>10.</td>
<td>12 = 2 × 2 × ____</td>
<td>30.</td>
<td>9 = 4 × 3 × 2 × ____</td>
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<tr>
<td>11.</td>
<td>12 = 3 × 2 × ____</td>
<td>31.</td>
<td>8 = 6 × 4 × ____ × 2</td>
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<tr>
<td>12.</td>
<td>20 = 5 × 2 × ____</td>
<td>32.</td>
<td>9 = 9 × 3 × ____ × 3</td>
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</tr>
<tr>
<td>13.</td>
<td>20 = 5 × 2 × ____</td>
<td>33.</td>
<td>8 = 8 × ____ × 8</td>
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<tr>
<td>14.</td>
<td>16 = 8 × ____</td>
<td>34.</td>
<td>7 = 7 × ____ × 7</td>
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<td>15.</td>
<td>16 = 4 × 2 × ____</td>
<td>35.</td>
<td>8 = 3 × ____ × 6</td>
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<td>16.</td>
<td>24 = 8 × ____</td>
<td>36.</td>
<td>16 = 2 × ____ × 4</td>
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<td>17.</td>
<td>24 = 4 × 2 × ____</td>
<td>37.</td>
<td>2 = 18 × ____ × 5</td>
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<tr>
<td>18.</td>
<td>24 = 4 × ____ × 2</td>
<td>38.</td>
<td>28 = 2 × ____ × 8</td>
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<tr>
<td>19.</td>
<td>24 = 3 × 2 × ____</td>
<td>39.</td>
<td>24 = 3 × ____ × 5</td>
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<tr>
<td>20.</td>
<td>24 = 3 × ____ × 2</td>
<td>40.</td>
<td>6 = 8 = ____ × 12</td>
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<tr>
<td>21.</td>
<td>6 × 4 = 8 × ____</td>
<td>41.</td>
<td>27 = 3 × ____ × 5</td>
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<tr>
<td>22.</td>
<td>6 × 4 = 4 × 2 × ____</td>
<td>42.</td>
<td>12 = 6 × ____ × 6</td>
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**EUREKA MATH**

**Lesson 8: Make equivalent fractions with the number line, the area model, and numbers.**

3/19/13

engageNYC

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Spend Some Time with 1 to 9

Problem-solving and practice to build fluency and number sense. www.Corelearn.com
Create Equations with the Digits 1–9

Create as many equations as you can with the digits 1 – 9:

• Use some or all of the digits in each equation.
  \[8 \div 4 = 5 - 3\] \[6 \times 7 = 42\] \[7 \times 5 + 8 - 6 = 29 + 1 + 3 + 4\]

• Do not use any digit more than once within any equation.
  \textit{non-example:} \[8 \div 4 + 3 = 7 - 3 + 1\]

• Do not use the digit zero.
  \textit{non-example:} \[16 \div 2 - 40 \div (8 - 3)\]

• You may use any math operations.
  \textit{Add, subtract, multiply, divide, exponents, etc.}
Mystery Math Grids

You try it. What are the outside numbers for this Mystery Math Grid?
Connections to Intervention and Special Education

• Research on priority

• Balance between conceptual and procedural (remember the five strands of proficiency)

• Application also builds fluency and number sense - Include reasoning and problem solving
Common Traits of Fluency + Number Sense Activities

• Build fluency
• Build number sense
• Doable – Quick, accessible, ”successible”
• Adaptable
• Durable
Common Issues with Fluency + Number Sense Activities

• Time constraints in lessons – fitting in fluency

• Finding fluency + number sense activities

• First few steps are the hardest
  • Teachers finding the right way to get students started
  • Teachers and students learning new activities and routines
Look-fors in Classrooms

• Short regular doses of fluency activities.
• Activities vary by intent – Fluency & Fluency+
• Activities vary by type – different types of fluency and different types of fluency+ activities
• Activities do not unintentionally take over lessons.
• Students are highly engaged. Not drill and kill, but rather strive and thrive.
• Questioning from teachers to get students to think
• Number lines
Favorite Fluency + Number Sense Activities

• Oral counting
• Card games
• Number Talks
• Mystery Math Grids
• Individual White Boards
• Spend Some Time with 1 to 9
• KenKen Puzzles (kenkenpuzzles.com)
• Sprints (Bill Davidson, EngageNY/Eureka Math)
Let’s Connect!

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corelearn.com

If you’d like CORE to come to your school to provide instructional coaching and other professional learning services, please get in touch.

Webinar Bonus!
Spend Some Time with 1 to 9
FREE samplers

www.corelearn.com/resources/free-materials