Mathematical Fluency – Techniques, Access and Sustainability for All Students

Presented by

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www.Corelearn.com

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Fluency – There is always your fingers…

Kid Snippets – "Math Class"

https://www.youtube.com/watch?v=KdxEAt91D7k

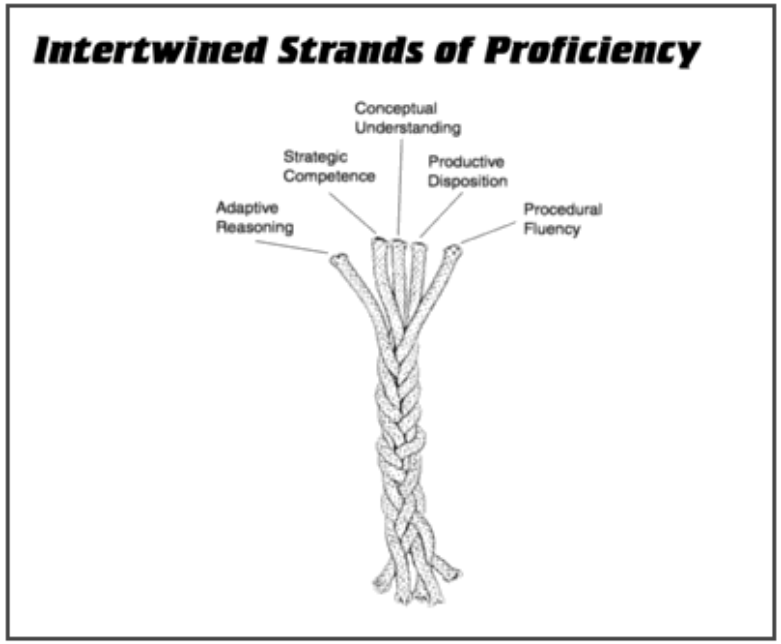
Essential Questions to Answer Today

- What is fluency and why is it important?
- What are different paths toward fluency?
- What are some ideas, activities and resources to implement in the classroom to build fluency, number sense, and reasoning skills?

Five Proficiency Strands in Mathematics

- Conceptual understanding
- Strategic competence
- Adaptive reasoning
- Productive disposition
- Procedural fluency

Box 4-1



National Research Council 2001

What Is Fluency?

. . . *procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately)* . . .

CCSSM, p. 6

Turn and talk with a partner:

- What does fluency mean to you?
- What are the types of things in math that students should be fluent with?

Fluency Chart for K–8

Grade	Required Fluency
K	Add and subtract within 5
1	Add and subtract within 10
2	Add and subtract within 20 (mentally) Add and subtract within 100
3	Multiply and divide within 100 Add and subtract within 1,000
4	Add and subtract multidigit whole numbers using standard algorithms
5	Multiply multidigit whole numbers using standard algorithm
6	Add, subtract, multiply, and divide multidigit numbers (including decimals) using standard algorithms
6-8	Compute with positive and negative fractions and decimals

Assessments and Fluency

Assessments typically include three key areas:

- Facts and procedures
- Concepts
- Applications

Review examples in handout (pp. 3-6)

Retaining Math Knowledge: Memory

Limits of Working Memory

$$7(x + 8) = 7x + 56$$

Capacity and Time

Age (Years)	Average Capacity Range (Chunks of Information)	Average Time Limit for Retention
Younger than 5	2 ± 1	Unknown
From 5 to 14	5 ± 2	5 to 10 minutes
Older than 14	7 ± 2	10 to 20 minutes

David A. Sousa, *How the Brain Learns Mathematics,* 2008

Practice Is Necessary

Studies show that if material is studied for one semester or one year, it will be retained adequately for perhaps a year after the last practice (Semb, Ellis, & Araujo 1993), but most of it will be forgotten by the end of three or four years in the absence of further practice. If material is studied for three or four years, however, the learning may be retained for as long as 50 years after the last practice (Bahrick 1984; Bahrick & Hall 1991).

Daniel Willingham, *American Educator*, Spring 2004

Rote and Elaborate Rehearsal

- **Rote rehearsal:** Memorization without continuing to think through an idea or fact.
- **Elaborate rehearsal:** Making sense of ideas and information. The learner processes and reprocesses information to connect it together, to connect it to prior learning, and to assign meaning to it. Elaborate rehearsal is necessary for students to probe the deeper meaning and interrelationships of mathematical concepts.

Why We Need Elaborate Rehearsal

Memories are formed as the residue of thought. You remember what you think about, but not every fleeting thought—only those matters to which you really devote some attention.

Daniel Willingham 2008

Fluency Activities with Number Sense in Mind

Paths to Fluency in Math

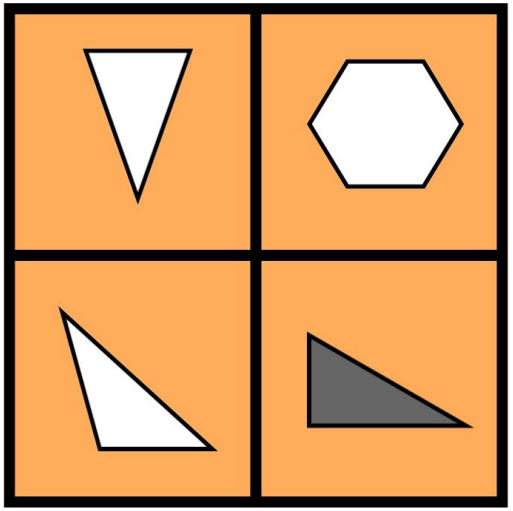
- Conceptual understanding
- Strategic Thinking
- Practice
- Excitement
- Problem-solving

Types of Activities

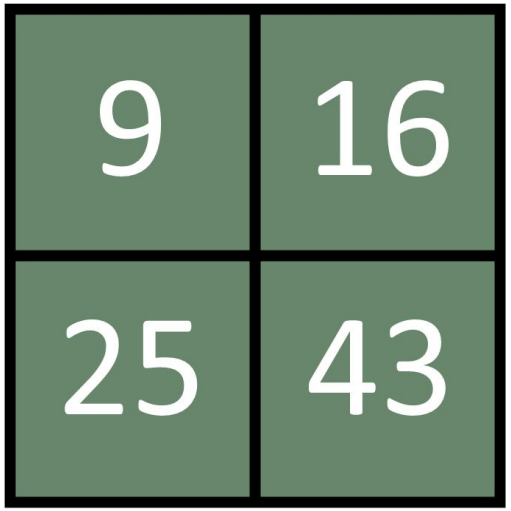
- Non paper and pencil
- Paper and pencil
- Online

Which One Doesn't Belong
wodb.ca

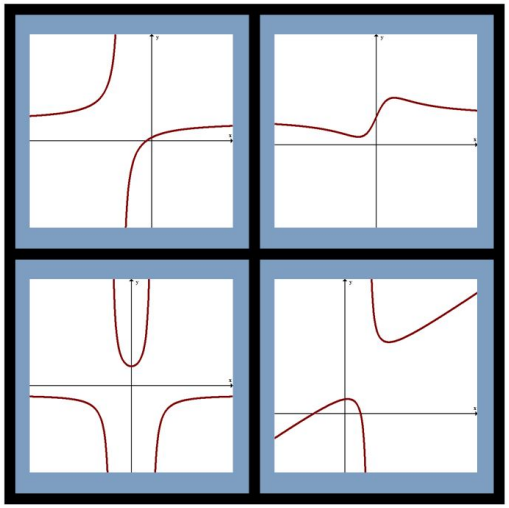
Shapes



Numbers



Graphs



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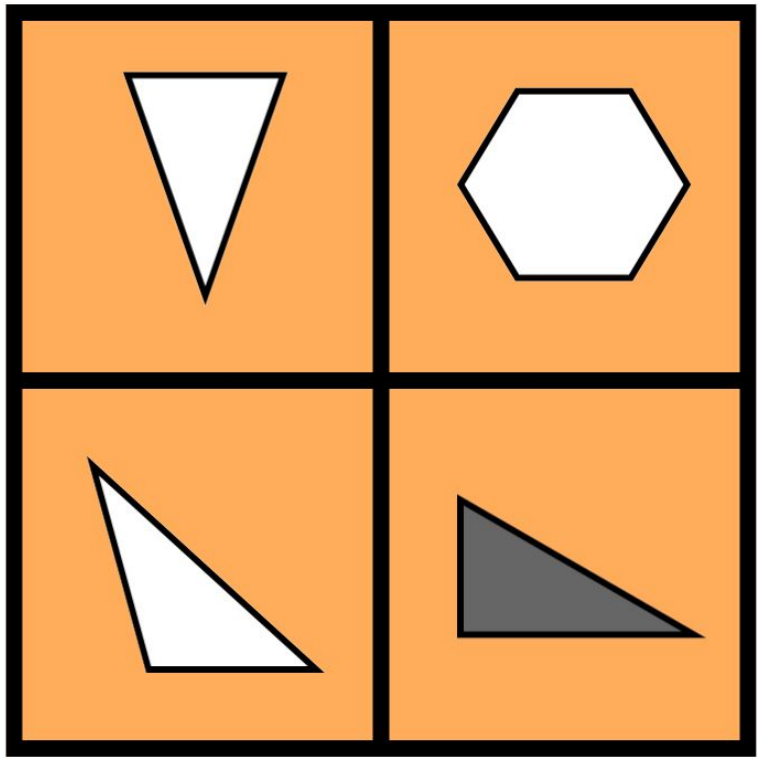
Shapes 1



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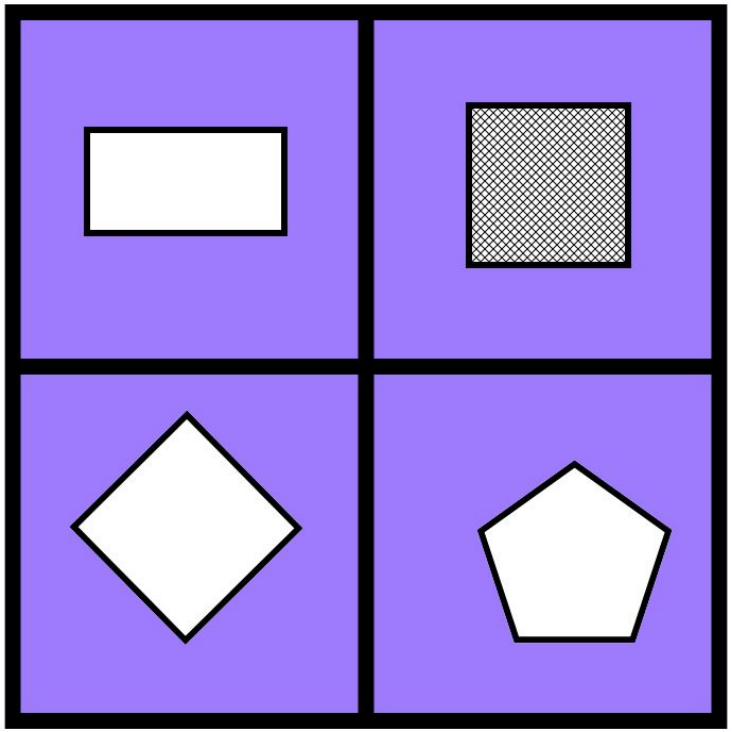
Shapes 2



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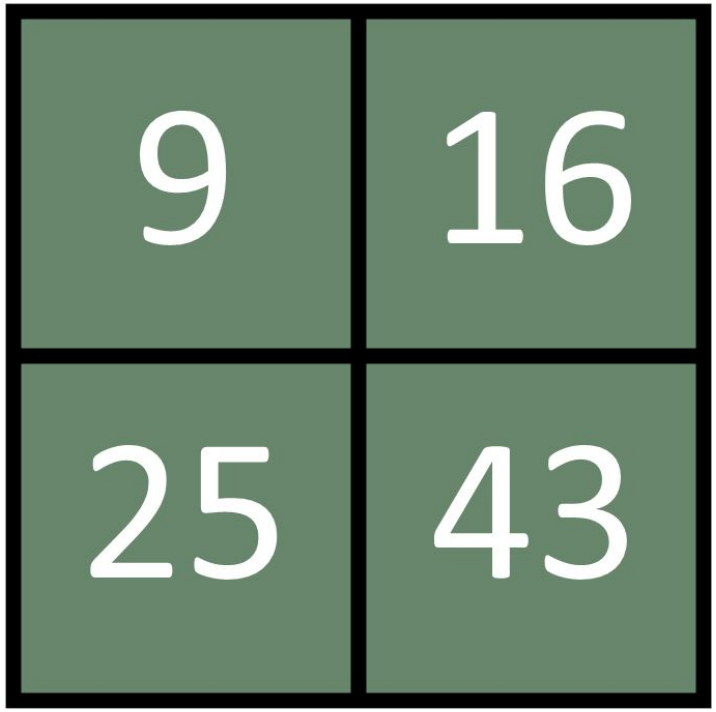
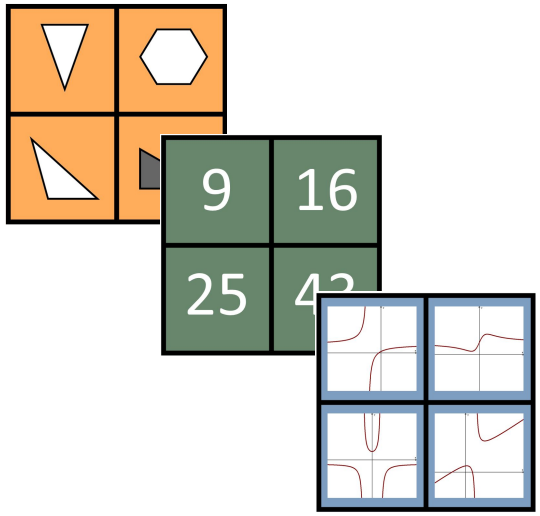


Which One Doesn't Belong - wodb.ca
Tips for Implementation



- Start with shapes
- Individual think time
- Confirm and connect
- Limit telling all you know
- Students explain in pairs and groups the **why**
- Progress to "finding a reason for each shape"

KenKenPuzzle.com

3+ 1	2	3 3
7+ 3	1	3+ 2
2 2	3	1

7+ 4	4+ 3	1	3+ 2
3	6+ 2	4 4	1
1 1	4	5+ 2	7+ 3
3+ 2	1	3	4

Counting Up and Down

- Count by twos starting at 8
- Count by twos again at 9
- Count by fives starting at 11

Advice:

- Start small
- Focus on where students are at to move forward
- Go back and forth across tens
- Discuss patterns and challenges
- Use a number line to model, display & build understanding
- Integers, fractions, decimals, exponents, algebraic expr.

Individual White Board Review

1. $\frac{1}{2} = \frac{\quad}{4}$

2. $\frac{1}{2} = \frac{\quad}{10}$

3. $\frac{1}{4} = \frac{\quad}{8}$

4. $\frac{1}{4} = \frac{\quad}{20}$

5. $\frac{3}{4} = \frac{\quad}{20}$

6. $\frac{1}{4} = \frac{3}{\quad}$

7. $\frac{3}{9} = \frac{\quad}{3}$

8. $\frac{4}{12} = \frac{2}{\quad}$

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Individual White Board Review

Advice:

- Review and build on important knowledge
- Plan problems ahead of time
- Decide on how students will share
- Scaffold and adjust on the fly as needed
- Stop, model, teach, & discuss as needed
- Limit time

Sprints – Practice not Punishment

- Practice of facts and skills
- Usually 44 problems
- Version A and B (matching pairs of Sprints)
- Problems designed in quadrants
- Take advantage of the math built into the Sprint
  - **Practice + Understanding**

Handout: 15-16

SPRINT

- Let's do a Sprint

It's not a race, it's not a test. It's practice, it's learning strategies and building fluency.

Sprints – Practice not Punishment

- Advice:
  - **EMPHASIZE – NOT GRADED, NOT A TEST**
  - **Look over page 8 in handout**
  - **Vary delivery**
  - **Match Sprints to student levels**
  - **Focus on improvement**
  - **Focus on understanding and practice**
  - **Don't have to give Sprints A and B on same day**

Spend Some Time with 1 to 9

- Problem-solving and practice to build fluency and number sense. **wwwCorelearn.com**

Create Equations with the Digits 1–9

Create as many equations as you can with the digits 1 – 9:

- Use some or all of the **digits** in each **equation**.

  $8 \div 4 = 5 - 3$ $\quad$ $6 \times 7 = 42$ $\quad$ $7 \times 5 + 8 - 6 = 29 + 1 + 3 + 4$

- Do not use any digit more than once within any equation.

  non-example: ~~$8 \div 4 + 3 = 7 - 3 + 1$~~

- Do not use the digit zero.

  non-example: ~~$16 \div 2 = 40 \div (8 - 3)$~~

- You may use any math operations.

  Add, subtract, multiply, divide, exponents, etc.

Mystery Math Grids

X	3	5	8
4	12	20	32
2	6	10	16
6	18	15	24

+	3	5	8
9	12	14	17
2	5	7	10
1	4	6	9

You try it. What are the outside numbers for this Mystery Math Grid?

X			
	6	21	24
	10	35	40
	12	42	48

Others

- Math Talks
- I Have You Have
- ArcAdemics.com
- Sumaze (app for phone/tablets)
- Desmos.com
- Fraction War, Fractions Pictionary, and Equivalent Fractions Musical Plates - Teachers Pay Teachers

  https://www.weareteachers.com/fun-with-fractions-7-tactile-and-kinesthetic-games/

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Booth #1009

Thank you!

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