CORE MATH®

Mathematical Fluency

Techniques, Access, and Sustainability for All Students

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Participant Handout
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Fluency Chart

Fluencies in the Common Core State Standards for Mathematics

Grade	Required Fluency
K	Add and subtract within 5
1	Add and subtract within 10
2	Add and subtract within 20 (mentally) Add and subtract within 100
3	Multiply and divide within 100 Add and subtract within 1,000
4	Add and subtract multidigit whole numbers using standard algorithms
5	Multiply multidigit whole numbers using standard algorithm
6	Add, subtract, multiply, and divide multidigit numbers (incl. decimals) using standard algorithms
6-8	Compute with positive and negative fractions and decimals

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Fluency Standards Corresponding to the Fluency Chart
(from the Common Core State Standards in Mathematics)

K.OA.5: Fluently add and subtract within 5.

1.OA.6: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

2.OA.2: Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

2.NBT.5: Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

3.OA.7: Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

3.NBT.2: Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

5.NBT.5: Fluently multiply multi-digit whole numbers using the standard algorithm.

6.NS.2: Fluently divide multi-digit numbers using the standard algorithm.

6.NS.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Grades 6-8: From page 84 in the CCSSM regarding transitions between MS and HS and from HS to College and Career there is indication of fluency from grades 6-8 with computing with rational numbers.

> *Indeed, some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems,* ***computing fluently with positive and negative fractions and decimals,*** *and solving real-world and mathematical problems involving angle measure, area, surface area, and volume.*

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Elementary School Samples

Consortium: SBAC Sample item ID: MAT.03.SR.1.00NBT.A.217	Standard: 3.NBT.2

The number sentence below can be solved using tens and ones. 67 + 25 = __?__tens and __?__ones. Select one number from each column to make the number sentence true.

Tens	Ones
○ 2	○ 2
○ 6	○ 5
○ 8	○ 10
○ 9	○ 12

Consortium: SBAC Sample item ID: MAT.03.SR.1.000OA.C.237	Standard: 3.OA.7

For items 1a–1c, choose Yes or No to show whether putting the number 7 in the box would make the equation true.

1a. 10 x ___ = 70 Yes __ No __

1b. 48 ÷ ___ = 6 Yes __ No __

1c. 63 ÷ ___ = 9 Yes __ No __

Consortium: PARC	Standard: 3.OA.7

Click on all the equations that are true.

- ☐ 8 x 9 = 81
- ☐ 54 ÷ 9 = 24 ÷ 6
- ☐ 7 x 5 = 25
- ☐ 8 x 3 = 4 x 6
- ☐ 49 ÷ 7 = 56 ÷ 8

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Middle School Samples

Consortium: SBAC Sample item ID: MAT.07.ER.3.0000G.B.160	Standard: 7.G.5, 6.EE.2

Part A Determine if each of these statements is always true, sometimes true, or never true. Circle your response.

1. The sum of the measures of two complementary angles is 90°.

Always True Sometimes True Never True

2. Vertical angles are also adjacent angles

Always True Sometimes True Never True

3. Two adjacent angles are complementary.

Always True Sometimes True Never True

4. If the measure of an angle is represented by x, then the measure of its supplement is represented by $180 - x$.

Always True Sometimes True Never True

5. If two lines intersect, each pair of vertical angles are supplementary.

Always True Sometimes True Never True

Part B For each statement you chose as "Sometimes True," provide one example of when the statement is true and one example of when the statement is not true. Your examples should be a diagram with the angle measurements labeled. If you did not choose any statement as "Sometimes True," write "None" in the work space below.

Consortium: SBAC Sample item ID: MAT. 07.SR.1.000EE.C.162	Standard: 7.EE.1, 7.EE.2

For numbers 1a–1e, select Yes or No to indicate whether each of these expressions is equivalent to $2(2x + 1)$.

1a. $4x + 2$	Yes __	No __
1b. $2(1 + 2x)$	Yes __	No __
1c. $2(2x) + 1$	Yes __	No __
1d. $2x + 1 + 2x + 1$	Yes __	No __
1e. $x + x + x + x + 1 + 1$	Yes __	No __

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Middle School Samples

Consortium: PARC	Standard: 6.NS.7b

The Tasty Treats Cake Factory bakes cakes to sell for a grocery chain. Each cake is weighed to see how close it is to the factory's target weight of 30 ounces. The scale shows how close the cake's weight is to the target. The scale will display:

- A positive number if the cake's weight is over 30 ounces.
- A negative number if the weight is less than 30 ounces.

The table shows two readings from the scale on Tuesday.

Cake	Reading
F	-5 oz.
G	-3 oz.

Which of the following statements is true?

A. Cake F weights less than Cake G because -5 < -3.
B. Cake F weights more than Cake G because -5 < -3.
C. Cake F weights less than Cake G because -3 < -5.
D. Cake F weights more than Cake G because -3 < -5.

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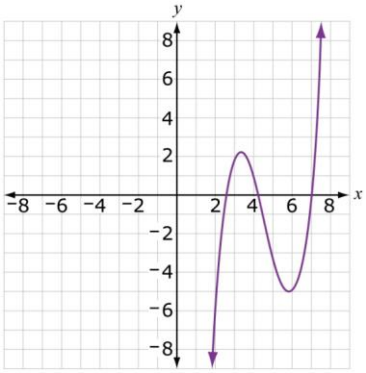
High School Samples

Consortium: SBAC Sample item ID: HS.SR.1.00FIF.K.082	Standard: F-IF.1

For numbers 1a – 1d, determine whether each relation is a function.

1a. {(0,1),(1,2),(3,1),(4,2)} Yes __ No __

1b. $y = \pm\sqrt{4 - x^2}$ Yes __ No __

1c. 

Yes __ No __

1d. {(5,3),(2,4),(5,2)} Yes __ No __

Consortium: SBAC Sample item ID: MAT.HS.SR.1.00NRN.A.152	Standard: N-RN.2

For items 1a – 1e, determine whether each equation is True or False.

1a. $\sqrt{32} = 2^{\frac{5}{2}}$ True __ False __

1b. $16^{\frac{3}{2}} = 8^2$ True __ False __

1c. $4^{\frac{1}{2}} = \sqrt[4]{16}$ True __ False __

1d. $2^8 = \left(\sqrt[3]{16}\right)^6$ True __ False __

1e. $\left(\sqrt{64}\right)^{\frac{1}{3}} = 8^{\frac{1}{6}}$ True __ False __

Consortium: PARC	Standard: A-REI.4

Solve the following equation: $(3x - 2)^2 = 6x - 4$

When you are finished, enter the solution(s) below.

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Math Fluency Activities and Deepening Number Sense

Types of Activities

No Paper or Pencil

- Counting Up and Down
- I Have - You Have
- White Boards
- Math Talks
- Fraction Folding
- *Others*

Paper and Pencil

- Mystery Math Grids
- Sprint
- Spend Some Time with 1 to 9
- *Others*

Online

- "Which one does not belong"
- KenKen Puzzles
- Sumaze
- ArcAdemics.com
- Desmos
- *Others*

Purposes for Fluency Activities

1. Build fluency
2. Build number sense
3. Maintenance
4. Preparation for current lesson
5. Anticipation of future lessons

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Techniques for Using Sprints for Both Fluency and Deepening Number Sense

- Preview the Sprint – give students 30+ seconds to look over but not write on the Sprint.
- Extended time – allow more time to complete a Sprint (1.5-3 minutes).
- Shorten the Sprint – fold it in half (hot-dog style) so students only focus on either the first half or the second half. If focused on the first half, then can save second half for another day, or include as the "extension" for those that finish the first half 'early'.
- Pick-one Do-one – students pick one problem from each quadrant of the Sprint and solve it, writing down the answer on the Sprint. This gives students a heads-up about what is on the Sprint and forces students to think about the problems, challenges with the problems, and which problem they would like to do ahead of time. This also gives students 2-4 problems already solved before the timed session begins.
- Solve all and record your time – students solve all the problems on either half, three-quarters, or the whole Sprint and record how long it took to solve it. Their goal is to beat this time on the second Sprint (which may be done the same day or another day).
- Preview at home – hand out the Sprint for some (those needing additional support) or all students to preview at home. If a Sprint is sent home, be sure to include clear and explicit instructions for what to do with it so parents are not confused. Explain that the last quadrant is meant to be challenge, so at home, perhaps only ask to look at the first three quadrants.
- Complete at home - hand out the Sprint for some (those needing additional support) or all students to complete at home. Then do the exact same Sprint in class.
- Do not have students do any type of 'preview" activity with Sprint A, but do something to identify patterns in Sprint A after completing Sprint A, or do a preview of Sprint B. This way students are more likely to all improve on Sprint B, and to see how seeing patterns and relationships helps build number sense.
- Secretly allow 10 extra seconds on Sprint B to increase likelihood of improvement.

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Counting Up and Down

- Use clear signals for counting up, counting down, and stop.
- Use a “stop” signal periodically as needed to help students reset.
- Recognize that counting down is a bigger challenge usually than counting up, however, counting down is equally meaningful.
- May start at different places, for example, “count by fives starting at four.”
- Don’t count with/for the students.
- Repeat counting across tens.
- Typically use 1, 2, 3 as in “one ten” “two tens” “three tens”, “one one-third, two one-thirds, three one-thirds”
- Have students lead these sometimes. Students can do counting activities in groups or pairs, with students being the “teacher”.
- Limit time (1-4 minutes) or students tire quickly of it. Can fit in anywhere in lessons.
- Intersperse reflection or other connection questions within a counting activity where appropriate. For example, counting by 50 cm, after doing several, ask, “how many cm in a meter?”
- Counting can include requirement to change units, or compose units. For example, when counting by 50 cm, when reaching a 100 cm mark, state amount in meters rather than centimeters.

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I Have – You Have

Write a target number on the board, for example, 10; and identify the operation used, for example, addition. Teacher says, “I have 3, you have?” Students respond “7”. Try to really animate it and the students really get into it.

Options:

- Have students say the number sentence, for example, three plus seven equals ten.”
- Do not identify an operation. Student 1 says “I have…”, “Student 2 responds to “You have…” and Student 3 determines the operation used.
- Have students play this in groups of three to five without the teacher. One of the students takes on the “I have…” role, and/or rotate the “I have…” around the small group.

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Mystery Math Grids

x	3	5	8
4	12	20	32
2	6	10	16
6	18	15	24

+	3	5	8
9	12	14	17
2	5	7	10
1	4	6	9

X	___	___	___
___	6	21	24
___	10	35	40
___	12	42	48

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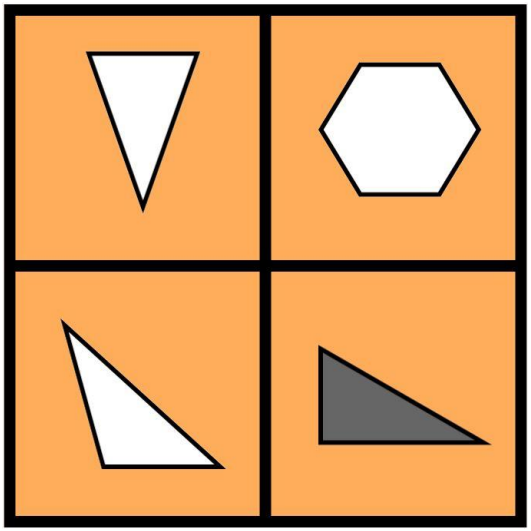
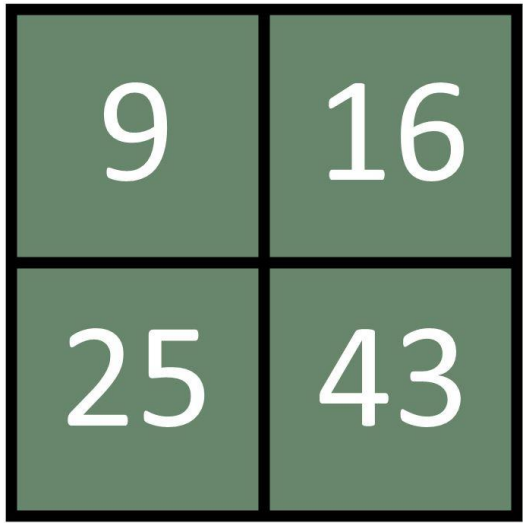
Which One Doesn't Belong

Wodb.ca

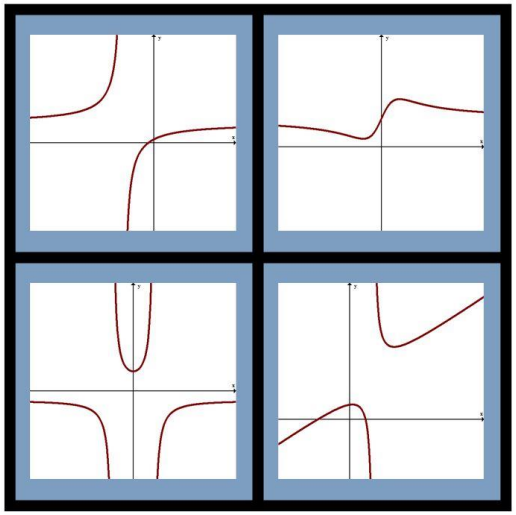
Shapes



Numbers



Graphs



Ken Ken Puzzles (kenkenpuzzle.com)

4+	7+		2
	3+		7+
6+	5+		
	2	4+	

1	6+		4+
5+	7+		
	3+		7+
7+			

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Puzzle No. 8352, 5X5, hard

12+	3—		11+	
	1—		6+	
	1—			4+
1—		7+		
	2—		9+	

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A # Correct _____

Find the missing numerator or denominator.

1	$\frac{1}{2} = \frac{_}{4}$		23	$\frac{1}{3} = \frac{_}{12}$	
2	$\frac{1}{5} = \frac{2}{_}$		24	$\frac{2}{3} = \frac{_}{12}$	
3	$\frac{2}{5} = \frac{_}{10}$		25	$\frac{8}{12} = \frac{_}{3}$	
4	$\frac{3}{5} = \frac{_}{10}$		26	$\frac{12}{16} = \frac{3}{_}$	
5	$\frac{4}{5} = \frac{_}{10}$		27	$\frac{3}{5} = \frac{_}{25}$	
6	$\frac{1}{3} = \frac{2}{_}$		28	$\frac{4}{5} = \frac{28}{_}$	
7	$\frac{2}{3} = \frac{_}{6}$		29	$\frac{18}{24} = \frac{3}{_}$	
8	$\frac{1}{3} = \frac{3}{_}$		30	$\frac{24}{30} = \frac{_}{5}$	
9	$\frac{2}{3} = \frac{_}{9}$		31	$\frac{5}{6} = \frac{35}{_}$	
10	$\frac{1}{4} = \frac{_}{8}$		32	$\frac{56}{63} = \frac{_}{9}$	
11	$\frac{3}{4} = \frac{_}{8}$		33	$\frac{64}{72} = \frac{8}{_}$	
12	$\frac{1}{4} = \frac{3}{_}$		34	$\frac{5}{8} = \frac{_}{64}$	
13	$\frac{3}{4} = \frac{9}{_}$		35	$\frac{5}{6} = \frac{45}{_}$	
14	$\frac{2}{4} = \frac{_}{2}$		36	$\frac{45}{81} = \frac{_}{9}$	
15	$\frac{2}{6} = \frac{1}{_}$		37	$\frac{6}{7} = \frac{48}{_}$	
16	$\frac{2}{10} = \frac{1}{_}$		38	$\frac{36}{81} = \frac{_}{9}$	
17	$\frac{4}{10} = \frac{_}{5}$		39	$\frac{8}{56} = \frac{1}{_}$	
18	$\frac{8}{10} = \frac{_}{5}$		40	$\frac{35}{63} = \frac{5}{_}$	
19	$\frac{3}{9} = \frac{_}{3}$		41	$\frac{1}{6} = \frac{12}{_}$	
20	$\frac{6}{9} = \frac{_}{3}$		42	$\frac{3}{7} = \frac{36}{_}$	
21	$\frac{3}{12} = \frac{1}{_}$		43	$\frac{48}{60} = \frac{4}{_}$	
22	$\frac{9}{12} = \frac{_}{4}$		44	$\frac{72}{84} = \frac{_}{7}$	

find the missing numerator or denominator

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B Improvement _____ # Correct _____

Find the missing numerator or denominator.

1	$\frac{1}{5} = \frac{2}{__}$		23	$\frac{1}{3} = \frac{4}{__}$	
2	$\frac{2}{5} = \frac{__}{10}$		24	$\frac{2}{3} = \frac{8}{__}$	
3	$\frac{3}{5} = \frac{__}{10}$		25	$\frac{8}{12} = \frac{2}{__}$	
4	$\frac{4}{5} = \frac{__}{10}$		26	$\frac{12}{16} = \frac{__}{4}$	
5	$\frac{1}{2} = \frac{2}{__}$		27	$\frac{3}{5} = \frac{15}{__}$	
6	$\frac{1}{3} = \frac{__}{6}$		28	$\frac{4}{5} = \frac{__}{35}$	
7	$\frac{2}{3} = \frac{4}{__}$		29	$\frac{18}{24} = \frac{__}{4}$	
8	$\frac{1}{3} = \frac{__}{9}$		30	$\frac{24}{30} = \frac{4}{__}$	
9	$\frac{2}{3} = \frac{6}{__}$		31	$\frac{5}{6} = \frac{__}{42}$	
10	$\frac{1}{4} = \frac{2}{__}$		32	$\frac{56}{63} = \frac{8}{__}$	
11	$\frac{3}{4} = \frac{6}{__}$		33	$\frac{64}{72} = \frac{__}{9}$	
12	$\frac{1}{4} = \frac{__}{12}$		34	$\frac{5}{8} = \frac{40}{__}$	
13	$\frac{3}{4} = \frac{__}{12}$		35	$\frac{5}{6} = \frac{__}{54}$	
14	$\frac{2}{4} = \frac{1}{__}$		36	$\frac{45}{81} = \frac{5}{__}$	
15	$\frac{2}{6} = \frac{__}{3}$		37	$\frac{6}{7} = \frac{__}{56}$	
16	$\frac{2}{10} = \frac{__}{5}$		38	$\frac{36}{81} = \frac{4}{__}$	
17	$\frac{4}{10} = \frac{2}{__}$		39	$\frac{8}{56} = \frac{__}{7}$	
18	$\frac{8}{10} = \frac{4}{__}$		40	$\frac{35}{63} = \frac{__}{9}$	
19	$\frac{3}{9} = \frac{1}{__}$		41	$\frac{1}{6} = \frac{__}{72}$	
20	$\frac{6}{9} = \frac{2}{__}$		42	$\frac{3}{7} = \frac{__}{84}$	
21	$\frac{1}{4} = \frac{__}{12}$		43	$\frac{48}{60} = \frac{__}{5}$	
22	$\frac{9}{12} = \frac{3}{__}$		44	$\frac{72}{84} = \frac{6}{__}$	

find the missing numerator or denominator

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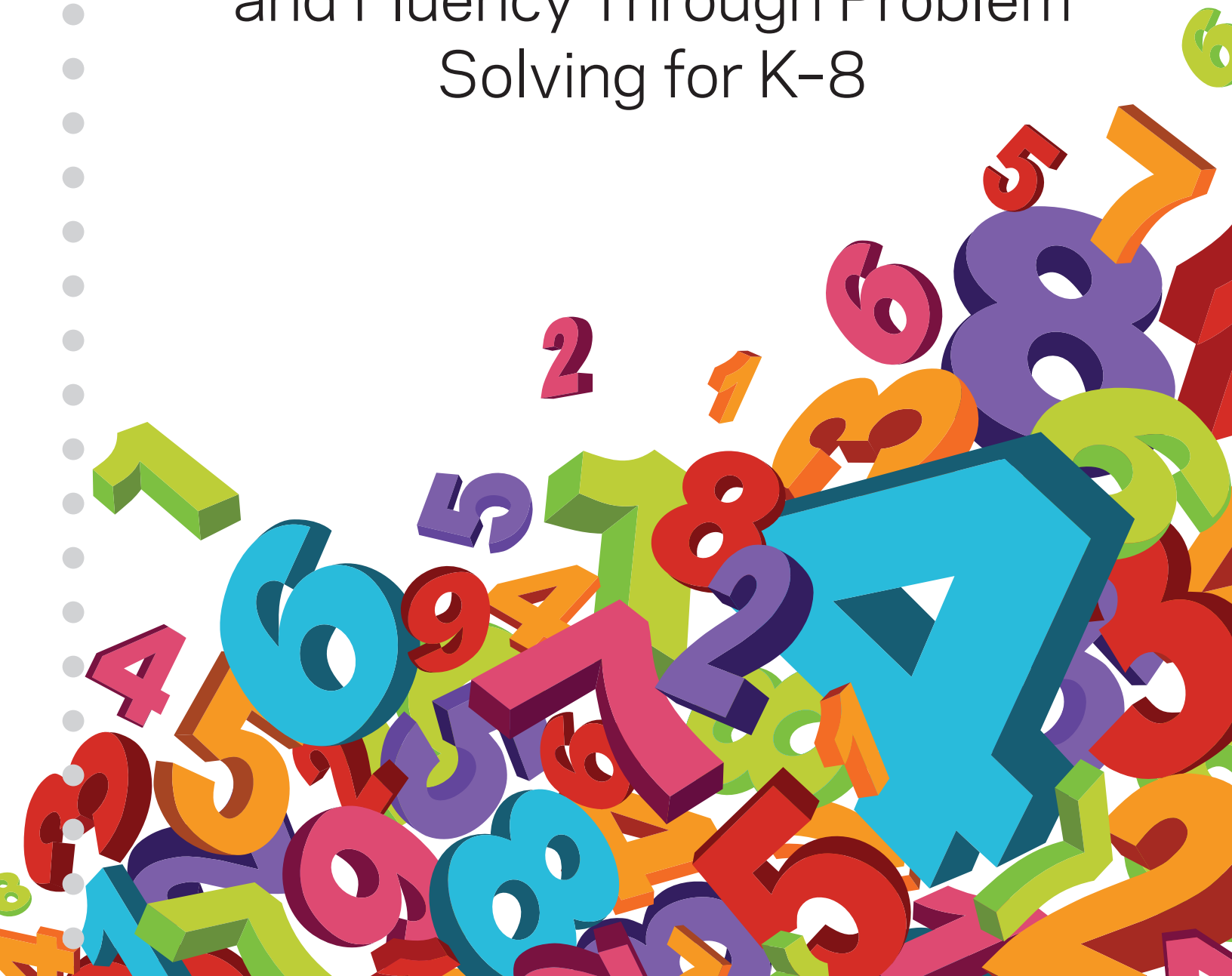
CORE MATH
NUMBER INTELLIGENCE

Spend Some Time with 1 to 9:

Building Number Sense and Fluency Through Problem Solving for K-8



Create Equations with the Digits 1–9

Create as many equations as you can with the following conditions:

- Use the digits 1–9 to create many different equations.
- Use some or all of the digits in each equation.
- Do not use any digit more than once within any single equation.
- Do not use the digit zero.
- You may use any math operation, including exponents.

For example:

$8 \div 4 = 5 - 3$ → uses the digits 3, 5, 4, and 8

$5 + 6 \times 4 = 29 \times 1$ → uses the digits 1, 2, 4, 5, 6, and 9

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Spend Some Radical Time with 1 to 9
Make the Inequality Statements True

1	3	5	7	9

1. Place **any** of the digits from the set above into the blank spaces in each inequality shown to the right to make the statement true.

   For example, below we have used 3, 5, and 7 to make a true statement:

   $$\boxed{3}\sqrt{2} < \sqrt{4\boxed{5}} < \sqrt{\boxed{7}8}$$

   - Do not use a digit more than once in the same statement.
   - Do not use a calculator.

a. $\square\sqrt{2} < \sqrt{4\square} < \sqrt{\square 8}$

b. $\sqrt{4\square} < \square\sqrt{2} < \sqrt{\square 8}$

c. $\sqrt{\square 8} < \sqrt{4\square} < \square\sqrt{2}$

2. Show at least two possible solutions for any problem that can have more than one solution.

d. $\sqrt{\square 8} < \square\sqrt{2} < \sqrt{4\square}$

3. If you were required to place the same number in each blank, is there any statement that is impossible to solve with this condition? If so, explain or prove why there is no possible solution in these cases.

e. $\sqrt{4\square} < \sqrt{\square 8} < \square\sqrt{2}$

f. $\square\sqrt{2} < \sqrt{\square 8} < \sqrt{4\square}$

4. What ideas or strategies did you use to help you solve some or all of these problems? Why do your ideas or strategies work?

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