

Challenges for EL Students to Overcome

—Adapted from California Math Framework, Universal Access, 2015 (30–32)

1. Limited prior or background knowledge and experience with formal schooling
 - May lack basic computation skills and/or have gaps in conceptual knowledge. EL students may be unfamiliar with learning habits expected in the classroom and for homework.
2. Cultural differences
 - May have different learning styles. May have different meanings for certain symbols such as commas and decimal points. Background knowledge may affect interpreting contexts and meanings for word problems, and expressions of commonly used applications for currency and measurements.
3. Linguistics
 - Challenges with understanding differences between academic language and everyday language, as well as math-specific terms and symbols. Included in this is the challenge of grasping the complex structure of the passive voice.
4. Polysemous words
 - Words with identical spellings and pronunciations, but different meanings that are based on context. For example, *table*, *operation*, *mean*, *even*, etc.
5. Word problems – syntactic features and text analysis
 - Word order is critical, and small words or expressions have precise meaning, such as *an* and *more than*. Word problems can have a complex syntax. Additionally, solving a word problem means understanding a context the problem is given in, translating words into English, identifying the question and relevant information, translating the English into math symbols and expressions, setting up and solving the math, and translating the solution into English.
6. Semantic features

As shown in the following table (adapted from NYU Steinhardt, 2009), many ELs may find semantic features challenging.

Feature	Examples
Synonyms	add, plus, combine, sum
Homophones	sum/some, whole/hole
Difficult expressions	If . . . then; given that . . .
Prepositions	divided into versus divided by; above, over, from, near, to, until, toward, beside
Comparative constructions	If Amy is taller than Peter, and Peter is taller than Scott, then Amy must be taller than Scott.
Passive structures	Five books were purchased by John.
Conditional clauses	Assuming ____ is true, then . . .
Language function words	Words and phrases used to give instructions, to explain, to make requests, to disagree, and so on

Identify Challenging Words

<p>Double Meanings: Words with a mathematical meaning and a nonmathematical meaning or connotation</p> <table style="width: 100%; border: none;"> <tr> <td style="width: 33%;">table</td> <td style="width: 33%;">fraction</td> <td style="width: 33%;">even</td> </tr> <tr> <td>base</td> <td>volume</td> <td>tangent</td> </tr> <tr> <td>line</td> <td>similar</td> <td>odd</td> </tr> <tr> <td>side</td> <td>rational</td> <td>variable</td> </tr> <tr> <td>point</td> <td>operation</td> <td>mean</td> </tr> <tr> <td>irrational</td> <td>expression</td> <td>term</td> </tr> <tr> <td>angle</td> <td>difference</td> <td>mode</td> </tr> <tr> <td>property</td> <td>face</td> <td>product</td> </tr> <tr> <td>shape</td> <td>altitude</td> <td>factor</td> </tr> <tr> <td>yard</td> <td>radical</td> <td>equal</td> </tr> </table>	table	fraction	even	base	volume	tangent	line	similar	odd	side	rational	variable	point	operation	mean	irrational	expression	term	angle	difference	mode	property	face	product	shape	altitude	factor	yard	radical	equal	<p>Homophones: Math words that have a non-math word with the same sound</p> <table style="width: 100%; border: none;"> <tr> <td>cent</td> <td>→</td> <td>sent or scent</td> </tr> <tr> <td>plane</td> <td>→</td> <td>plain</td> </tr> <tr> <td>two</td> <td>→</td> <td>to or too</td> </tr> <tr> <td>sum</td> <td>→</td> <td>some</td> </tr> <tr> <td>sine</td> <td>→</td> <td>sign</td> </tr> <tr> <td>four</td> <td>→</td> <td>for or fore</td> </tr> <tr> <td>whole</td> <td>→</td> <td>hole</td> </tr> <tr> <td>eight</td> <td>→</td> <td>ate</td> </tr> <tr> <td>pi</td> <td>→</td> <td>pie</td> </tr> <tr> <td>one</td> <td>→</td> <td>won</td> </tr> <tr> <td>weigh</td> <td>→</td> <td>way</td> </tr> <tr> <td>ten</td> <td>→</td> <td>tin</td> </tr> </table>	cent	→	sent or scent	plane	→	plain	two	→	to or too	sum	→	some	sine	→	sign	four	→	for or fore	whole	→	hole	eight	→	ate	pi	→	pie	one	→	won	weigh	→	way	ten	→	tin
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<p>Multiple Terms for Same Idea: Words in math that overlap, or are often used interchangeably</p> <ul style="list-style-type: none"> • altitude, height or length • base, width or length • add, sum • solve, determine the answer, compute • justify, explain, prove • multiply, determine the product 	<p>Small Words or Phrases: Words or phrases that students often struggle to interpret in math problems</p> <table style="width: 100%; border: none;"> <tr> <td>or</td> <td>fewer</td> <td>increase</td> </tr> <tr> <td>many</td> <td>then</td> <td>and</td> </tr> <tr> <td>less than</td> <td>decrease</td> <td>left</td> </tr> <tr> <td>more than</td> <td>an</td> <td>as many as</td> </tr> <tr> <td>of</td> <td>greater</td> <td>all</td> </tr> <tr> <td>any</td> <td>off</td> <td>take away</td> </tr> </table>	or	fewer	increase	many	then	and	less than	decrease	left	more than	an	as many as	of	greater	all	any	off	take away																																																
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<p>Unique Terms: Words that have meaning only in math</p> <p>hypotenuse parallelogram coefficient quadratic</p>	<p>Similar Sounding Words:</p> <p>tens vs. tenths then vs. than sixty vs. sixteen</p>																																																																		

Addressing the Needs of EL Students in Mathematics

Recommendations for Connecting Mathematical Content to Language

1. Focus on students' mathematical reasoning, not accuracy in using language.
2. Focus on mathematical discourse practices, not language as words, or grammar.
3. Recognize the complexity of language in math classrooms.
4. Treat everyday language as a resource, not as an obstacle.
5. Uncover the mathematics in what students say and do.

—Moschkovich, 2012 (5–8)

Support English learners as they learn both mathematics and academic language

- Explicitly teach and incorporate into regular practice academic vocabulary for math. Be aware of words that have multiple meanings such as *root*, *plane*, or *table*.
- Provide communication guides, sometimes called sentence frames, to help students express themselves not just in complete sentences, but articulately within the MP standards.
- Use graphic organizers and visuals to help students understand mathematical processes and vocabulary.
- Elementary school English learners' progress in mathematics may be supported through the intentional lesson planning for content, mathematical practice, and language objectives. Language objectives "...articulate for learners the academic language functions and skills that they need to master to fully participate in the lesson and meet the grade-level content standards."

—Echevarria, Short, & Vogt, 2008 (55)

The following are examples of possible language objectives for a student in grade 2:

- *Read addition and subtraction expressions fluently.*
- *Explain the strategies and/or computational estimates used to solve addition and subtraction problems within 100.*
- *Describe the relationship between multiplication and division.*

Integrating the CA ELD Standards into K–12 Mathematics and Science Teaching and Learning

at: <http://www.cde.ca.gov/sp/el/er/eldstandards.asp>.

Developed by WestEd

How the document is organized

The *Integrating the CA ELD Standards into K–12 Mathematics and Science Teaching and Learning* document describes how each of the 19 ELD standards are addressed through specific examples within each of four grade bands (K-2, 3-5, 6-8, and 9-12). The document consists of a series of tables for each grade band, organized around sets of ELD standards. The tables are color coded. For example, the first set of tables for each grade band are shaded purple. The purple tables include descriptions of the four ELD standards in Part I-A: *Interacting in Meaningful Ways – Collaborative (engagement in dialogue with others)*.

Each table in the document displays the grade band, the ELD standards, alignment to the Standards for Mathematical Practice (SMPs) that are linked to the CA ELD Standards, and specific examples from lessons illustrating alignment to the specific CA ELD standard.

As you read the examples it will become clear that most instructional techniques designed to help EL students are good techniques for all students. A list of the typical teacher moves or teaching techniques used in various forms throughout the document are shown in the chart below. In the chart, techniques are organized under general categories such as *Discourse*, *Visual Aids*, and *Grouping Strategies*.

English Learner Proficiency Levels

- **Emerging:** Students at this level typically progress very quickly, learning to use English for immediate needs as well as beginning to understand and use academic vocabulary and other features of academic language.
- **Expanding:** Students at this level are challenged to increase their English skills in more contexts and learn a greater variety of vocabulary and linguistic structures, applying their growing language skills in more sophisticated ways that are appropriate to their age and grade level.
- **Bridging:** Students at this level continue to learn and apply a range of high level English language skills in a wide variety of contexts, including comprehension and production of highly technical texts. The “bridge” alluded to is the transition to full engagement in grade-level academic tasks and activities in a variety of content areas without the need for specialized ELD instruction.

Integrated and Designated ELD

- **Integrated ELD** – Build and support English literacy while learning content. The content is the focus.
- **Designated ELD** – Protected time for EL students with focus on language instruction, utilizing core content material and the CA ELD Standards.

Check this out: YouTube: *Three Minute Thriller on Integrated and Designated ELD*
<https://www.youtube.com/watch?v=ozGB4ItMHHE>

Common Techniques Used to Address English Learner Needs

A	B	C	Techniques
			DISCOURSE (student discussion and writing about math)
			<ul style="list-style-type: none"> • Construct good questions that promote critical thinking and extended discourse.
			<ul style="list-style-type: none"> • Sentence starters and sentence frames.
			<ul style="list-style-type: none"> • Think-pair-share and think-pair-share-write.
			<ul style="list-style-type: none"> • Restate or re-voice student explanations using appropriate math terminology, and/or the structure and language features of mathematical descriptions.
			<ul style="list-style-type: none"> • Prompting a student to elaborate on a response to extend his or her language use and thinking.
			<ul style="list-style-type: none"> • Paraphrasing a student's response and including target academic language as a model and, at the same time, accepting the student's response using every day or "flawed" language.
			<ul style="list-style-type: none"> • Linking what a student is saying to prior knowledge or to learning that will come (previewing).
			VISUAL AIDS
			<ul style="list-style-type: none"> • Using a range of information systems, such as graphic organizers, diagrams, photographs, videos, or other multimedia, to enhance access to content, such as, Word Walls, Anchor Charts, Number Lines, and Frayer charts.
			GROUPING STRATEGIES
			<ul style="list-style-type: none"> • Providing a variety of collaborative groups, such as, <ul style="list-style-type: none"> ○ Think-pair-share and think-pair-share-write ○ Pair Emerging and Expanding students with upper Expanding, Bridging and proficient students in their own language to discuss, compare, and write.
			ORGANIZING LESSONS
			<ul style="list-style-type: none"> • Construct good questions that promote critical thinking and extended discourse.
			<ul style="list-style-type: none"> • Select and sequence tasks, such as modeling and explaining, and providing guided practice, in a logical order
			<ul style="list-style-type: none"> • Frequently check for understanding during instruction, as well as gauging progress at appropriate intervals throughout the year. Adjust instruction appropriately based on formative assessments.
			<ul style="list-style-type: none"> • Choose texts carefully for specific purposes (e.g., motivational, linguistic, content)
			EL SPECIFIC STRATEGIES
			<ul style="list-style-type: none"> • Pair Emerging and Expanding students with upper Expanding, Bridging and proficient students in their own language to discuss, compare, and write.
			<ul style="list-style-type: none"> • Teacher works with small groups of EL (Emerging and Expanding) students to clarify by reading together, translating, illustrating, paraphrasing, collaboratively creating and/or analyzing, etc.
			<ul style="list-style-type: none"> • Allowing and encouraging students to first talk and/or write in their primary language and then translate this communication into English.

References

California English Language Development Standards (2014), California Department of Education, Sacramento, CA., <http://www.cde.ca.gov/sp/el/er/eldstandards.asp#Standards>

CA ELD Standards into K-12 Mathematics and Science Teaching and Learning (2015), California Department of Education with WestEd, <http://www.cde.ca.gov/sp/el/er/eldstandards.asp>.

CORE Word Knowledge Chart

1. Record the word in the top row.
2. Write a definition/meaning of the word.
3. Show examples of the word.
4. Make a drawing that represents the word.
5. Record other words that you associate with this word or characteristics of the word.

Word	
<i>Fraction</i>	
<p>Meaning</p> <p><i>Part of a whole.</i></p> <p><i>Write it as one number over the other number.</i></p> <p><i>One number divided by another number.</i></p>	<p>Visualization or Drawing</p> <p>$\frac{2}{5} =$ </p>
<p>Example</p> <p>$\frac{2}{5}, \frac{7}{4}$</p>	<p>Other Characteristics or Related Words</p> <p><i>numerator</i></p> <p><i>denominator</i></p> <p><i>improper fractions</i></p>

CORE Word Knowledge Chart

Word	
Meaning	Visualization or Drawing
Example	Other Characteristics or Related Words

Symbol and Cue Cards

1. Create a set of *symbol cards* and matching *cue cards* as shown on the following pages.
 - **Symbol cards:** Cards containing math terms, expressions, equations, etc.
 - **Cue cards:** Cards containing phrases that match one or more of the symbol cards. You may have more than one cue card for the same symbol card.
2. Play the game:
 - a. Give each student (or pair of students) a set of symbol cards.
 - b. Have students match symbol cards to cue cards by one of the following methods. Always check for understanding (correct matching). Students can also be required to record matches.
 - i. Option 1: Someone (teacher) reads a cue card and students hold up the corresponding symbol card.
 - ii. Option 2: Hand out a set of the cue cards to each student (or pair of students) and have students match cards.

Symbol Cards

$8 - n$	$n < 8$	$8 + n$
$8n$	$n > 8$	$8(n)$
$n - 8$	$n = 8$	$n + 8$
$8 / n$	$n / 8$	$\frac{n}{8}$

Cue Cards

The product of n and 8	8 is greater than n	The sum of 8 and n
The product of 8 and n	8 is less than n	The sum of n and 8
8 less than n	n is greater than 8	The difference between 8 and n
n less than 8	n is less than 8	The difference between n and 8
8 more than n	The quotient of 8 divided by n	8 out of n equal parts
n more than 8	The quotient of n divided by 8	n out of 8 equal parts
8 divided by n	8 n 's	n eighths
n divided by 8	8 times n	n and 8 are the same

Symbol and Cue Cards

1. Create a set of *symbol cards* and matching *cue cards*.
 - **Symbol cards:** Cards containing math terms, expressions, equations, etc.
 - **Cue cards:** Cards containing phrases that match one or more of the symbol cards. You may have more than one cue card for the same symbol card.
2. Play the game:
 - a. Give each student (or pair of students) a set of symbol cards.
 - b. Have students match symbol cards to cue cards by one of the following methods. Always check for understanding (correct matching). Students can also be required to record matches.
 - i. Option 1: Someone (teacher) reads a cue card and students hold up the corresponding symbol card.
 - ii. Option 2: Hand out a set of the cue cards to each student (or pair of students) and have students match cards.

Symbol Cards

Cue Cards

Best Deal

One store is having a 50% off sale. Another store has a 40% discount, with an additional 15% off of the sale price. Which sale should you take advantage of if you want the best reduction on a sweater that costs \$68.79?

Individual and Pair Work:

- **Individually:** Guess which is the better deal. Record.
- **Pair-share:** Share guess and reasoning.
- **Individually:** Solve the problem. Show your work.
- **Pair-share:**
 - Explain what you would have to pay in each sale.
 - Compare similarities/differences between the two sales.
 - Share your solution and justification.

Language Scaffolds

- I think the 50% discount is _____ than the 40% plus an added 15% discount off of the sale price because _____.
- Other sentence frames:
 - The store with the best reduction is _____ because _____.
 - The sales at the two stores are similar because they both _____.
 - The sales at the two stores are different because _____.
 - Comparing the sales at the two stores is tricky because _____.

Concept Sort

1. Divide students into pairs.
2. Give each pair of students a set of concept sorting cards (see the example page below for *Polygon Cards for Polygon Sort Activity* handout).
3. Students cut out the cards.
4. Students sort the cards/concepts into groups based on similar traits or properties.
5. Students explain the reasoning behind the groupings. It is recommended that each student record his/her work on an activity sheet such as the one provided in the sample Concept Sort Activity for polygons.
 - Although the activity sheet has space for sorting the cards into as many as five groups, students are not expected to sort the cards into exactly five groups. They may sort them into two to four groups, or in rare cases more than five groups.

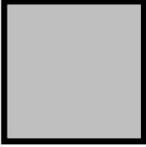
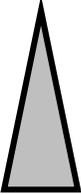
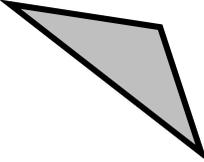
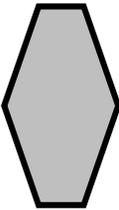
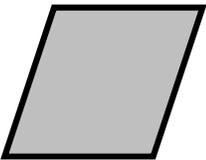
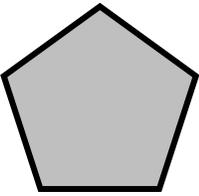
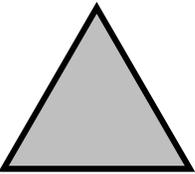
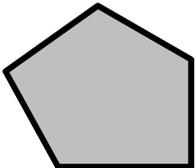
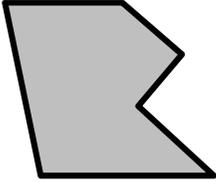
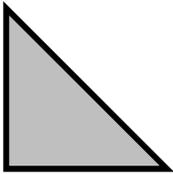
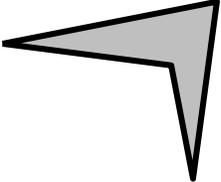
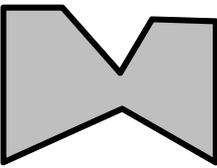
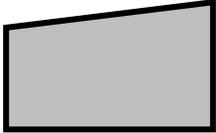
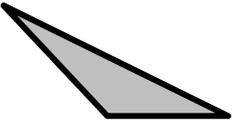
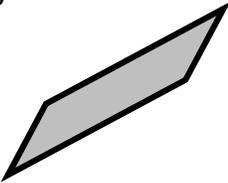
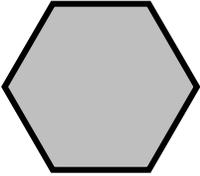
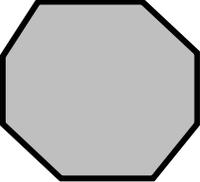
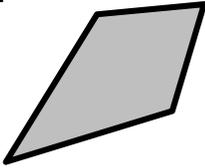
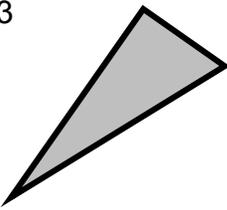
Extension 1:

1. Students leave the concepts organized into the groups they have chosen at their desk/table.
2. Pairs from different table groups switch places and analyze the organization and grouping of another pair of participants. They are not to see each other's activity sheets describing the grouping criteria.
 - The new pair should record the criteria they believe were used by the original pair of students to make the groupings.
 - Check with the other group to verify the criteria.

Extension 2:

1. Pairs trade one of their activity sheets with another pair (the activity sheets should have an explanation for how the cards were sorted, but not a listing of which cards are sorted into which groups). Each pair will do the following:
 - Use the other pair's sorting criteria to re-sort their own set of cards.
 - Check with the original group/pair to see if their sorting matches that from the original pair of students.
 - Discuss whether the sorting criteria described on the activity sheet were complete and distinct.
 - Complete: Were all cards accounted for?
 - Distinct: Did each card fit in exactly one group, or did one or more cards fit into more than one of the groups?

Polygon Cards for Polygon Sort Activity

1 	2 	3 	4 
5 	6 	7 	8 
9 	10 	11 	12 
13 	14 	15 	16 
17 	18 	19 	20 
21 	22 	23 	24 