Discourse in Math — Don't Just Talk About It

By Dean Ballard

Learning mathematics is not a spectator sport. The rigorous mathematical knowledge sought for at all levels of instruction requires deep thinking and persistent sense making from students. Communication about mathematics among students and between students and the teacher is the vehicle for bringing thinking to the surface, clarifying ideas, moving ideas forward, revealing misconceptions, and making key mathematical connections clear, transferable, and memorable. Mathematical discourse is the verbal and written communication that is centered around deepening thinking about and making sense of mathematics. Brummer and Kartchner Clark (2014) state, "students must think about, read about, talk about, and write about information in order to synthesize it and to retain it" (p. 21). Students cannot learn only by being told or shown information. Through language students communicate in ways that engage them in reasoning and talking about math (Fogelberg et al., 2008; McKee & Ogle, 2005). The math standards of all states emphasize the importance of student communication of mathematical ideas, making mathematical discourse a required process in learning mathematics.

Facilitating meaningful mathematical discourse . . . What are **teachers** doing?

Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.

(NCTM, 2014)

Discourse Through Mathematical Discussions

One of the most important mediums for communication is talking about math, what is often referred to as discourse. Discourse is not just any talk about math. Meaningful discourse includes students comparing and contrasting ideas and methods, constructing viable arguments, critiquing each other's reasoning, and helping each other make sense of mathematics. (Hattie, Fisher, & Frey, 2017; National Council of Teachers of Mathematics [NCTM], 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers [CCSSM], 2010; National Research Council, 2001). When focused and properly facilitated, discourse provides a powerful means for increasing understanding about mathematical ideas and concepts. Well facilitated discourse begins with good questions or an engaging and focused task, is managed by the teacher to stay focused on the learning objective, and concludes with the key mathematical connections made clear to all students.

Facilitating meaningful mathematical discourse . . . What are **students** doing?

Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole class discourse.

(NCTM, 2014)

Asking Deep-Level Questions

Meaningful discussions are built on deeper-level questions and higher-level tasks (NCTM, 2014; Smith & Stein, Smith, Henningsen, & Silver 2000). In a meta-analysis of educational research described in the Institute of Education Sciences Practice Guide, *Organizing Instruction and Study to Improve Student Learning,* Pashler et al. (2007) found there to be "strong" evidence (their highest level) for the positive effects of asking deep-level questions, questions that "prompt students to reason about underlying explanatory principles" (pp. 29-31). Deeper level questions are most often linked to tasks that are at Webb's (2017) depth of knowledge (DOK) levels 2 and 3. While instruction must include tasks that target factual knowledge (DOK 1), it must also include higher-level tasks targeting more rigorous understanding (DOK levels 2 and 3). Tasks at these levels engage students in connecting mathematical ideas, facts, and procedures, building conceptual understanding and applying mathematics to solve non-routine problems (Hess, Carlock, Jones, & Walkup, 2010; Woodward et al., 2012). Often DOK 1 tasks can be transformed into higher-level tasks by using deeper level questions. Questions such as those that follow are good at initiating reasoning at DOK levels 2-3 (Consortium on Reaching Excellence in Education [CORE], 2017, p. 4; Hess et al., 2010):

How would you compare ____ with ____?

Can you predict the outcome if ____?

What is the best method and why?

The types of tasks and questions we provide students are the foundation for productive discourse.

Moving Discourse Forward

Simply asking a deeper-level question or providing a higher-level task does not automatically lead to meaningful discussions. Getting students to think and respond to questions and guiding those discussions to the mathematical objectives of the lesson requires ongoing facilitation from the teacher (Hattie et al., 2017). This facilitation begins with initiating discourse and continues through promoting, managing, connecting and concluding the discourse so that important mathematical connections are made explicit to all students. Arbaugh (2010) describes several key ideas to keep in mind to successfully facilitate this process.

> Research indicates that teachers support their students' classroom conversations by (a) giving students sufficient time to work on and discuss worthwhile mathematical problems, (b) pressing for justifications and explanations, (c) refraining from thinking for the students, and (d) encouraging students to learn from one another by modeling productive thinking. (pp. 45-46)

Talk Moves for Promoting and Managing Discourse

- Think time
- Wait time
- Revoice
- Restate
- Revise
- Add on
- Pair-share
- Reason about
- Think-write-pair-share
- Think-pair-share-write

Managing, Processing, and Concluding Discourse

To turn a meaningful prompt into a meaningful discussion teachers must employ effective talk moves, such as, think time, wait time, turn-and-talk, think-pair-share, and think-write-pair-share. Moves such as revoicing and adding-on help scaffold discussions and extend ideas shared by students. To deepen learning teachers can ask additional questions that have students reason about or compare shared ideas, or answer "what if" questions. Additionally, the classroom environment must be one that encourages students to safely express both right and wrong ideas. When wrong answers are valued as opportunities for learning students feel greater freedom to express their ideas. All of the talk moves mentioned above depend on students being willing to discuss and share their thinking.

Finally, the teacher facilitates making the mathematical connections explicit by using guiding questions and suggestions, modeling processes and strategies, periodically processing and checking answers, and explicitly pulling ideas together at the end of the discussion (Hattie et. al, 2017; Hiebert & Grouws, 2007; NCTM, 2014; Parrish & Dominick, 2016). Below is a CORE chart that summarizes ideas for initiating, managing, and concluding discourse.

Strategies to Initiate, Manage, and Connect Discourse (CORE, Inc., 2015)

I. Initiate Discourse

1. Ask students engaging and challenging questions. For example,
   - Simply ask, "Why?" Ask compare and contrast or agree/disagree questions.
   - Ask "How does this connect to previous work?" questions.
   - Provide sentence frames when needed to scaffold beginning discussions.
   - Ask questions that have students build mathematical concepts one upon another. For example, after reviewing subtraction of whole numbers on a number line ask, "What do you think it means if we subtract 5 from 3?" (starting negative numbers).
2. Provide think and discussion time. Use turn-and-talk, think-pair-share, and think-write-pair-share. Discourse is student-to-student, whole class and student-to-teacher.
3. Let students know what is expected, what to think and write about (not what to think). Tell students this may be different from prior experiences and expectations in math classes.

II. Manage Discourse

1. Begin with small expectations in terms of discussion time and provide time frames for students. For example, "Class, you have 30 seconds to decide if you agree or disagree with Jerry's answer and why. Then you will have two minutes to discuss this with your partner and be ready to explain your answer to the class."
2. Circulate and listen in on conversations while students talk.
3. While circulating during student-to-student discussions, be prepared to guide, refocus, and move past blockages in discussions as needed by asking questions to help scaffold ideas or making suggestions to discussions. Leave room for students to think. Do not do all the thinking for them.

III. Connect Discourse

1. Choose students to share using selective methods to have specific ideas shared that you overheard, or use the "cold call" method to encourage all students to be ready to share.
2. Have students share various strategies to a solution.
3. Correct misconceptions as needed. Invite students into this process.
4. Build on student ideas whenever possible. Connect student ideas or ask students to connect ideas that are shared. (Repeat, revise, restate, and add-on strategies.)
5. Make the mathematical ideas explicit. Clarify and validate correct mathematical thinking. Connect ideas, methods, thinking, and/or conclusions to the learning objective(s).

Discourse Protocols

There are several research based protocols teachers can employ to help facilitate focused and productive discourse. Smith and Stein (2011, p. 8) describe five practices that can be used to facilitate productive movement from small-group to whole-class discussion:

1. *Anticipating* likely student responses to challenging mathematical tasks
2. *Monitoring* students' actual responses to the tasks
3. *Selecting* particular students to present their mathematical work
4. *Sequencing* sharing students' responses in a specific order for discussion
5. *Connecting* different students' responses and connecting the responses to key mathematical ideas

These five practices provide a sequence of steps for instruction that keep the focus on the mathematics. At the same time these steps move conversations from individuals to a whole-class setting where key connections and concepts are made explicit by the teacher and by students. Another classroom discussion methodology is called "Accountable Talk" from the Institute for Learning at the University of Pittsburgh. Accountable Talk is focused on three areas of accountability (Michaels, O'Connor, Williams Hall, & Resnick, 2010):

1. *Accountability to the Learning Community*
2. *Accountability to Accurate Knowledge*
3. *Accountability to Rigorous Thinking*

Talk is initiated with challenging tasks, and students are held accountable to discuss the mathematics at deeper levels. Accountable Talk includes being respectful and active listeners, and trying to be accurate in what is said, not just saying the first thing that comes to mind.

Number Talks is also an excellent discourse technique. Parrish and Dominick describe Number Talks as "five- to fifteen-minute classroom conversation[s] around purposefully crafted problems that are solved mentally" (p. 13). For example, the teacher begins a Number Talk by asking a class of 4th grade students to decide which is greater, 5/8 or 8/20. Students are not allowed to write or use calculators and are encouraged to use multiple methods to solve the problem. Students first think about the problem individually, then share ideas with a partner, and finally the teacher selects ideas to be shared and discussed publicly with the whole class.

Writing Is an Important Form of Discourse

In addition to talking, writing is an important form of discourse in mathematics (NCTM, 2014). In *Teach Like a Champion 2.0,* Doug Lemov (2015) concludes that "the amount and quality of writing students do in your classroom are two of the most important determinants of their academic success" (p. 281). Important outcomes from writing include increased engagement when every student writes and greater understanding from more focused thinking. In *Visible Learning for Mathematics,* John Hattie et al. (2017) state, "The first (and possibly easiest) layer of accountability you can offer students is to ensure that if anyone writes, everyone writes. . . . Something about writing helps to focus the mind" (p. 163). Not all students can speak at once; however, all students can write at the same time and therefore, be thinking about the mathematics. Through writing students have the opportunity to individually brainstorm, deepen understanding through explanation and justification, and self-reflect on learning. Additionally, it provides teachers greater opportunity to assess learning and provide strategic feedback. Murry (2004, pp. 88-89) describes the following benefits of writing:

Benefits of Writing

- Thoughtfulness and increased reasoning skills
- Active involvement in thinking and making sense
- Questions raised and new ideas explored
- Use of higher-order thinking while interpreting and explaining data
- Clarification, reinforcement, and deepened conceptual understanding
- Teacher insights [formative assessment and strategic feedback]
- Self-reflection and self-directed learning

Teachers should use a variety of writing activities that link content knowledge with understanding and that require students to think deeply about mathematics. The same questioning techniques described earlier, questions that promote thinking at DOK levels 2-3, typically work well as writing prompts. Assigning students to write about their thinking before discussion helps students organize their thoughts and empowers students to share and discuss these ideas with others. Writing after discussion or instruction holds students accountable to personally process, reflect on, and understand what was said. Writing gives every student the opportunity to take ownership of math knowledge and make important connections that lead to better understanding.

Make Discourse Meaningful

Whether using one of the protocols described above, the talk moves earlier described, or another sequence of steps for discourse, it is important to recognize discourse is only effective if it is well managed. All techniques for discourse emphasize the need to begin with something worth talking or writing about, use prompts that guide students on how to target their discussion, provide time and processes to promote meaningful conversations, and conclude the discourse using clear means to make the desired mathematical connections explicit. Meaningful discourse can precede work on a math problem, be part of the work on a problem, or follow on the heels of individual time spent working on or thinking about a problem. Discussions can be short turn-and-talk events that retain the essential elements of Accountable Talk, or be longer discussions that utilize in some form the five practices for facilitating discourse from Smith and Stein (2011). The effectiveness of discourse is not based on the length of a discussion or the amount of writing done, but in what is being communicated. Students must be sharing about mathematical connections, meaning, applications, comparisons, conjectures, and justification (*why* rather than *what*). In *Adding It Up* the National Research Council (2001, pp. 426-426) provides several important recommendations for meaningful discourse of any length:

- A significant amount of class time should be spent in developing mathematical ideas and methods rather than only practicing skills.
- Questioning and discussion should elicit students' thinking and solution strategies and should build on them, leading to greater clarity and precision.
- Discourse should not be confined to answers only but should include discussion of connections to other problems, alternative representations, solution methods, and the nature of justification and argumentation, and the like.

In *Teach Like a Champion 2.0,* Doug Lemov (2015) describes two important "ratios" that need to occur simultaneously in classrooms, the *participation ratio* and the *think ratio* (pp. 234-235). The *participation ratio* is a ratio of students' active participation in the lesson, while the *think ratio* is about the level of rigor in the lesson. Full participation with low rigor is no better than low participation with high rigor. We need high ratios of participation by students and high levels of rigor in what we are asking from students. Lemov goes on to identify three key areas for teaching and learning that build both ratios: questioning, writing, and discussion. Discourse, both in talking about and writing about mathematical ideas, is critical for revealing, deepening, and expanding mathematical knowledge.

Discourse at Work — a Sample from the Classroom

A major part of CORE's work with schools takes place in the classroom and one aspect of this work is providing demonstration or model lessons. Demonstration lessons are one of the most influential methods for supporting teachers' reflection on instructional practices (Strayer, et al., 2017). With model lessons teachers and other observers first preview the lesson, then observe the lesson as it is taught by the CORE expert, and finally debrief the lesson. Through collaboration with the CORE expert, teachers plan how to integrate the ideas from the lesson into their own instruction.

The following demonstration lesson provided teachers an example of how discourse plays a critical role in students' mathematical learning and modeled facilitation practices that lead to making discourse meaningful. The lesson focused on how to compute the volume of rectangular solids, and transfer that knowledge to computing volumes of other shapes. The lesson was taught by a CORE expert who is referred to as the teacher. School site teachers observed the lesson and debriefed immediately after the lesson.

The Demonstration Lesson

At the start of the lesson the students had a general idea of the term *volume* from seeing containers filled with rice. The lesson began with a review of the meaning of *volume* referencing students' prior experience. Following this, the teacher facilitated a process of students determining the volumes of small boxes, then medium size boxes, then large boxes, and finally any rectangular solid. Students began by filling small boxes manually with cubes which evolved to them using rulers to determine how many cubes would be needed to fill a large box. The keys for meaningful discourse revolved around providing specific prompts that connected to each stage of the lesson.

- *How many cubes does it take to fill the small box? How do you know?*
- *How many cubes are in one complete layer of your medium-size box? How do you know? How many layers would it take to fill your box and how do you know?*
- *Explain how could you figure out the total number of cubes needed to fill the large box if all you had was a ruler but no cubes.*

Throughout the lesson students worked and discussed ideas in pairs based on the prompts. Sometimes writing preceded discussions and sometimes it followed discussions. In their responses, students were required to use key math terms such as *volume, base,* and *area of the base.* Students included a diagram and labeled parts of the diagram. Writing and discussion times were limited to a few minutes each in order to keep the lesson flowing for all students. Selected students shared ideas with the whole class (sometimes the teacher pretended to cold-call on these students). The teacher clarified and connected the ideas shared, keeping in mind the learning objectives. Through the lesson, students reinforced and deepened their understanding of volume, learned that they could compute the volume of a rectangular box by *length x width x height* and understood the general formula for computing volumes of all prisms and cylinders, *area of base x height.*

Classroom discourse played a major role in advancing student thinking. Students shared, compared, refined, and corrected their methods. Writing was instrumental at first in students collecting, processing, and furthering their own thoughts before sharing with another student, and finally as a process to record and organize thinking after discussion with a partner and the whole class. With guidance from the teacher, providing focused prompts, discussion and writing were instrumental in developing, deepening, and solidifying key mathematical knowledge.

The Approach of the Consortium on Reaching Excellence in Education

Coaching teachers on implementing effective discourse in the classroom through collaboration, observation and reflection, and model lessons like the volume lesson described above are part of the extensive professional learning services provided by CORE. CORE's experience in schools and classrooms confirms the finding in research: one of the greatest needs in math instruction is that of fostering student engagement to increase both the participation and think ratios.

For over 20 years, CORE has provided technical assistance and professional development to the most vulnerable schools in America. CORE's model rests on the research on effective professional learning that fosters durable implementation. CORE's approach builds knowledge and skills through well-structured courses and site-based coaching, modeling, and mentoring. CORE's Math Academies (Elementary, Middle and High School) and other math trainings blend theoretical and practical knowledge with hands on practice, video models, collaborative discussions, reflection, readings, simulations, modeling and feedback from an expert — the hallmarks of quality professional learning. CORE's typical implementation model emphasizes ongoing job-embedded coaching to implement new ideas and techniques in the classroom.

Job-Embedded Coaching and Support

CORE's job embedded coaching and site support moves from knowledge learning to transfer and application. During on-site days, the CORE specialist models best practices using the curricula and materials the teachers have. Teachers meet with the CORE expert individually and in grade groups to debrief lessons and then practice together to refine their techniques. In addition, CORE experts support the teachers and administrators as they study student data, collaboratively problem solve and plan instructional interventions, and gain additional practice and coaching. CORE experts guide administrators and site coaches on learning walks to study implementation and identify areas requiring further practice and support. During the course of a year, the CORE expert mentors the site leaders and coaches as they develop their own coaching and facilitative skills, so that they build on the work of the external expert in order to sustain implementation efforts for the long haul.

Conclusion

Initiating, managing, and concluding discourse is a multifaceted sequence that requires planning and effort throughout the lesson by the teacher. Whether for brief moments of discourse or extended discussions, to be meaningful, verbal and written communications must be focused, and mathematical connections made explicit. As teachers become more expert in facilitating this process, students will become more expert in mathematics.

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