Course Title: Online Math Academy – Fractions Course 2: Multiplication and Division with Fractions

Class Type: 6-module asynchronous course; 2 to 3 hours per module, over 4 weeks.

Course Description

This asynchronous course on Fractions is a highly interactive robust course looking deeply at the teaching and learning of fractions. In this course participants will deepen their knowledge about fraction concepts, connect physical and visual models to number lines and numerical procedures with operations with fractions, learn strategies to address common misconceptions, reflect and share ideas with colleagues, and continue to deepen knowledge for teaching fraction concepts and procedures through further guided study and applications. By engaging in this course, participants will gain the understanding necessary to deliver evidence-based instruction that develops mathematical proficiency with fractions. This deep dive into the Number-Operations-Fraction domain will increase expertise and confidence with leading students to a deep understanding of fractions that is important for success with more complex mathematics.

Each module in this course provides multiple formats for learning including video, text, problem-solving, hands-on manipulative activities, online interactive resources, discussions, and opportunities to reflect on your own. Course content provides a balance between acquiring information and application of new learning. In these course participants will explore conceptual, procedural declarative knowledge and problem-solving applications of fractions while modeling and discussing best practices. Each module integrates best practices taken from the most current research including modeling, language supports, connections to curricula, and connections to big ideas.

Educators take this course with a cohort of their peers who want to learn effective standards-aligned and evidence-based practices for teacher about fractions with all learners. Participants are expected to have regular access to computers and proficiency with email and current internet browsers. The course facilitator will answer questions, monitor work and progress, provide feedback, and monitor and add onto course discussion boards.

Learning Outcomes

At the conclusion of this course, participants will be able to:

1. Understand the progression of fraction concepts through multiple grade levels.
2. Recognize the connections between whole number and fraction concepts and procedures.
3. Recognize the connections within fraction concepts and procedures.
4. Identify, connect, and understand how to use multiple representations of fractions (concrete, visual/representational and abstract) to teach fraction concepts and operations.
    - Multiplication with fractions
    - Division with fractions

5. Access physical, digital, and visual models of fractions as means to make fraction concepts clear for students.
6. Understand strategies to address common misconceptions with fractions.
7. Connect content standards and the Standards for Mathematical Practices to the teaching and learning of fractions.
8. Explain the recommendations and rationale from research for best practices in teaching about fractions.
9. Apply learning to their own teaching (or coaching).

Teaching/Learning Strategies

- Working through and studying online explanation and examples.
- Watching video clips utilizing online resources to illustrate examples for teaching.
- Completing interactive learning activities with built in feedback.
- Completing weekly Journal reflections.
- Completing self-check "quizzes" with immediate feedback.
- Participating in weekly online discussions with peers.
- Developing further learning through independent assignments of choice.
- Conferencing with the course instructor.

Optional Reading

Elementary and Middle School Mathematics Teaching Developmentally (Van de Wall, Karp and Bay-Williams, any edition from 2007 - 2019 will be sufficient, 6th edition through 10th edition). Within each homework is a reading **option**. If participants choose to do the reading option of the assigned homework with each module, they will need access to the Van de Wall, Karp and Bay-Williams book. The price of the book is not included in the cost of the course since work in the book is optional. The book can be rented on Amazon or directly from Savvas for approximately $40 or purchased for about $80.

College Credit Option

Participants may receive 1 graduate-level semester extension unit for the course for an additional cost of $109 per. These units are typically used for salary advancement. There are no additional requirements beyond successful completion of this 16-hour course.

See Course Outline on the following pages.

Fraction Course 2 – Multiplication and Division with Fractions

Week 1:
- Module 1: Orientation and Introduction
- Module 2: Multiplication with Part 1

Week 2:
- Module 3: Multiplication with Fractions Part 2

Week 3:
- Module 4: Division with Fractions Part 1

Week 4:
- Module 5: Division with Fractions Part 2
- Module 6: Course Wrap Up

Module 1: Course Orientation and Introduction	
The course orientation is designed to help you get to know (or review) some basics about online learning in general and this course in particular. You will learn about the layout of the course, how to navigate the course, and learn about and experience several types of activities that are used throughout the course. The course introduction is designed to orient you to the course, review recommendations from research for the teaching and learning of fraction concepts and procedures.	
Sections in Module to Read and Study	**Related Activities to Complete**
Complete the *Orientation* section. • Orientation Intro • Tips for Online Learning • About this Course • Types of Pages and Assignments • Packet: Types of Pages in the Course	• Video – Course Introduction • Short Answer – Question about a Book • Multiple Choice - Homework Preference • Matching – Identify Course Icons • Discussion Forum: Introduce Yourself • Orientation Survey
Complete the *Introduction* section. • Welcome • Course Objectives • Design of the Course • Rationale for the Course	• Video – Course Introduction • Anticipation Guide • Short Answer - “What can you say about 6/8?”
Complete the *Practices and Proficiencies* section. • Five Proficiency Strands in Mathematics • Standards for Mathematical Practice • Critical Topics in Fractions	• Matching - Proficiency Strands with Instructional Episodes • Journal – Share learning from Dana Center Videos on Math Practices • Journal – Identify fraction progression and key vocabulary in your own state standards

Module 2: Multiplication with Fractions Part 1

In this module, you will:

- Connect physical and visual models to number line and numerical procedures for multiplying fractions.
- Work through activities that highlight a progression of learning with multiplying fractions.
- Learn best practices for teaching procedures with fractions.
- Share and discuss ideas related to teaching fraction multiplication.

The big mathematical ideas (Van de Walle, Karp, and Bay-Williams 2019) that are the focus of Module 2 are:

- The meanings of each operation with fractions are the same as the meanings for the operations with whole numbers. Operations with fractions should begin by applying these same meanings to fractional parts.
- Repeated addition and area [and other] models support development of concepts and algorithms for multiplication of fractions.

(pp. 373-74)

Sections in Module to Read and Study	Related Activities to Complete
Complete the *Introduction* section. • Objectives and Big Ideas • Discussion about Challenges with Teaching Fraction Multiplication • Iterating and Partitioning • Progression of Instruction	• Discussion Forum – Challenges with Teaching Fraction Multiplication • Matching – Iterating and Partitioning with Examples • Journal – Progression in Your Curriculum
Complete the *Multiplying: Whole Number x Fraction* section. • Fraction Multiplied by a Whole Number with Physical Models • Fraction Multiplied by a Whole Number with Visual Models	• Discussion Forum – Explore and Share on Uses of Pattern Blocks • Video – Modeling Multiplication with Counters • Short Answer – Describe Physical Models You Use
Complete the *Multiplying: Fraction x Whole Number* section. • Whole Number Multiplied by a Fraction with Physical Models • Whole Number Multiplied by a Fraction with Visual Models • Whole Number Multiplied by a Fraction as Scaling • Multiplying Whole Numbers and Fractions Activity	• Video – Modeling ¾ × 5 on a Number Line • Discussion Forum –Physical Models • Video – Multiplying Fraction x Whole Number with Cubes • Short Answer – Visual Models • Matching – Physical, Visual, Numerical
Complete the *Multiplying Fractions and Whole Numbers and Vice Versa* section. • Commutative Property • Multiplying Whole Numbers and Fractions Reflection	• Short Answer – Commutative Property • Journal – Multiplying Whole Numbers and Fractions

Module 3: Multiplication with Fractions Part 2

In this module, you will:

- Connect physical and visual models to number line and numerical procedures for multiplying fractions.
- Work through activities that highlight a progression of learning with multiplying fractions.
- Learn different approaches to teaching about multiplication with mixed numbers.
- Learn best practices for teaching procedures with fractions.
- Share and discuss ideas related to teaching fraction multiplication.
- Extend knowledge through classroom application or further study.

The big mathematical ideas (Van de Walle, Karp, and Bay-Williams 2019) that are the focus of Module 3 are:

- The meanings of each operation with fractions are the same as the meanings for the operations with whole numbers. Operations with fractions should begin by applying these same meanings to fractional parts.
- Repeated addition and area [and other] models support development of concepts and algorithms for multiplication of fractions.
- Estimation should be an integral part of computation development to keep students' attention on the meanings of the operations and the expected sizes of the results.

(pp. 373-74)

Sections in Module to Read and Study	Related Activities to Complete
Complete the *Multiplying Fractions with Fractions* section • No Subdividing with Unit Fractions • No Subdividing with Non-unit Fractions • Subdividing with Unit Fractions • Subdividing – Unit Fractions with Area Models • Discovering the Algorithm with Unit Fractions	• Short Answer – Identify Common Features • Video – Modeling with Cuisenaire Rods • Upload Work – Model Solving Problems • Short Answer – Comparing Problems • Video – Modeling with Paper Folding • Short Answer – Paper Folding • Short Answer – Importance of Context • Short Answer – Describing Patterns
Complete the *Multiplication* section. • From Area Models to Algorithms with Non-unit Fractions • From Area Models to Algorithms Review	• Video – Modeling with a Number Line • Discussion Forum – Prompts for Students to Develop the Algorithm • Discussion Forum – Addressing Common Errors
Complete the *Mixed Numbers* section. • Estimation with Mixed Numbers • Multiplying Mixed Numbers – Two Methods	• Short Answer – Estimating with Mixed Numbers
Complete the *Challenge* section • Estimation with Word Problems • Matching Problems, Processes, and Equations	• Matching – Estimated Answers for Word Problems • Matching – Answers for Word Problems

Complete *the Wrap Up and Homework* section • Wrap Up • Homework	• Journal – Modules 2-3 reflection **Homework:** A. Option A: Apply learning to classroom instruction and reflect on outcomes. B. Option B: Read/share from *Elementary and Middle School Mathematics*. C. Option C: Analyze Your Curriculum to connect to ideas from this module.

Module 4: Division with Fractions Part 1

In this module, you will:

- Connect representations of fraction division with physical and visual models and on a number line to numerical procedures.
- Connect conceptual knowledge to procedures.
- Understand why conceptual understanding is key to applying fraction division to solving real world problems.
- Learn how to use language to highlight the meaning of division.
- Learn different techniques for dividing fractions.
- Learn best practices for teaching the procedures for dividing with fractions.
- Work through activities that highlight a progression of learning with dividing fractions.
- Share and discuss ideas related to teaching fraction division.
- Extend knowledge through classroom application or further study.

The big mathematical ideas (Van de Walle, Karp, and Bay-Williams 2019) that are the focus of Module 5 are:

- The meanings of each operation with fractions are the same as the meanings for the operations with whole numbers. Operations with fractions should begin by applying these same meanings to fractional parts.
- Partition [partitive] and measurement [quotative] models lead to two different thought processes for division of fractions.
- Estimation should be an integral part of computation development to keep students' attention on the meanings of the operations and the expected sizes of the results.

(pp. 373-74)

Sections in Module to Read and Study	Related Activities to Complete
Complete the *Introduction* section. • Objectives and Big Ideas • Division with Fractions is Complicated	• Discussion Forum – Challenges with Fraction Division

Complete the *Key Prior Knowledge* section. • Relationship Between Multiplication and Division • Partitive and Quotative Division • Problem Types with Division • Identify Problem Types	• Short Answer – Fact Families • Multiple Choice – Partitive or Quotative • Multiple Choice – Identify Problem Types
Complete the *Five Techniques for Dividing with Fractions* section. • Overview of Five Techniques • Divide the Numerators and the Denominators • Common Denominators • Use Patterns to Identify Short Cuts • Multiply by the Reciprocal • Two-Step Process: Multiply by the Denominator and Divide by the Numerator	• Short Answer – Identifying Patterns 1 • Short Answer – Identifying Patterns 2 • Multiple Choice – Identify the Division Method
Complete the *Whole Number and Unit Fraction Division* section. • Unit Fractions divided by Whole Numbers • Whole Numbers divided by Unit Fractions • Dividing a Whole Number by a Unit Fraction as Partitive Division • Whole Number and Unit Fraction Division Wrap Up	• Video – Unit Fractions Divided by Whole Numbers • Matching – Word Problems with Partitive and Quotative 1 • Video – Whole Numbers Divided by Unit Fractions as Partitive Division • Matching – Word Problems with Partitive and Quotative 2 • Journal – Key Take-aways

Module 5: Division with Fractions Part 2

In this module, you will:

- Connect representations of fraction division with physical and visual models and on a number line to numerical procedures.
- Connect conceptual knowledge to procedures.
- Understand why conceptual understanding is key to applying fraction division to solving real world problems.
- Learn how to use language to highlight the meaning of division.
- Learn different techniques for dividing fractions.
- Learn best practices for teaching the procedures for dividing with fractions.
- Work through activities that highlight a progression of learning with dividing fractions.
- Share and discuss ideas related to teaching fraction division.
- Extend knowledge through classroom application or further study.

The big mathematical ideas (Van de Walle, Karp, and Bay-Williams 2019) that are the focus of Module 5 are:

- The meanings of each operation with fractions are the same as the meanings for the operations with whole numbers. Operations with fractions should begin by applying these same meanings to fractional parts.
- Partition [partitive] and measurement [quotative] models lead to two different thought processes for division of fractions.
- Estimation should be an integral part of computation development to keep students' attention on the meanings of the operations and the expected sizes of the results.

(pp. 373-74)

Sections in Module to Read and Study	Related Activities to Complete
Complete the *Whole Number Divided by a Non-unit Fraction* section. • Whole Number ÷ Non-unit Fraction = Whole Number Quotient • Whole Number ÷ Non-unit Fraction = Non-Whole Number Quotient	• Short Answer – Identify Commonalities in Three Word Problems • Short Answer – Making Sense of a Procedural Step • Short Answer – Making Sense of the Cups of Sugar for Muffins Problem
Complete the *Non-unit Fraction Divided by a Whole Number* section. • Non-unit Fraction Divided by a Whole Number • How Much in One Group? • Word Problems	• Short Answer - Identify Commonalities in Three More Word Problems • Video – Non-Unit Fraction Divided by a Whole Number • Short Answer - Identify Commonalities in Another Set of Three Word Problems • Upload File – Solving a Word Problem
Complete the *Non-Unit Fraction Divided by a Unit Fraction* section • Non-Unit Fraction ÷ Unit Fraction with the Same Denominator • Non-Unit Fraction ÷ Unit Fraction with a Different Denominator **Complete the *Any Fraction Divided by Any Fraction* section**	• Video – Non-Unit Fraction Divided by a Unit Fraction • Short Answer – Explaining Division

• Any Fraction ÷ Any Fraction • Other Considerations with Fraction Division **Complete *the Wrap Up and Homework* section** • Wrap Up • Homework	• Video – Proof of the Standard Division Algorithm Using Known Properties • Matching – Estimating Quotients • Short Answer – Mental Math – Order of Values of Division Expressions • Journal – Module reflection **Homework:** A. Option A: Apply learning to classroom instruction and reflect on outcomes. B. Option B: Read/share from *Elementary and Middle School Mathematics*. C. Option C: Analyze Your Curriculum to connect to ideas from this module.

Module 6: Course Wrap Up – Reflection, Connecting and Shifting Practice	
In the final module of the course, you will: • Revisit the goals and objectives of the course. • Consider how new learning from this course will shift your practice. • Review and reflect on the methods for developing the Standards for Mathematical Practice in students. • Explore additional online resources for fraction activities.	
Sections in Module to Read and Study	**Related Activities to Complete**
Complete the *Reflecting* section. • Revisiting the Goals and Objectives • Progression of Critical Standards • Importance of the Standards for Mathematical Practice **Complete the *Shifting Practice* section.** • Fraction Splats! • Farewell and Good Teaching • Additional Resources	• Video – Course Summary • Journal – Revisiting the Anticipation Guide • Journal – Describing Key Take-aways • Journal – Connections to SMPs • Exploration – Explore Fraction Splats • Video – Farewell Message • Explore – Explore Additional Online Resources for Work with Fractions

Course Resources and References

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