Course Title: Online Math Academy – Fractions Course 1: Fraction Meaning, Equivalence, Addition and Subtraction

Class Type: 7-module asynchronous course; 1 to 3 hours per module, over 4 weeks.

Course Description

This asynchronous course on Fractions is a highly interactive robust course looking deeply at the teaching and learning of fractions. In this course participants will deepen their knowledge about fraction concepts, connect physical and visual models to number lines and numerical procedures with operations with fractions, learn strategies to address common misconceptions, reflect and share ideas with colleagues, and continue to deepen knowledge for teaching fraction concepts and procedures through further guided study and applications. By engaging in this course, participants will gain the understanding necessary to deliver evidence-based instruction that develops mathematical proficiency with fractions. This deep dive into the Number-Operations-Fraction domain will increase expertise and confidence with leading students to a deep understanding of fractions that is important for success with more complex mathematics.

Each module in this course provides multiple formats for learning including video, text, problem-solving, hands-on manipulative activities, online interactive resources, discussions, and opportunities to reflect on your own. Course content provides a balance between acquiring information and application of new learning. In these course participants will explore conceptual, procedural declarative knowledge and problem-solving applications of fractions while modeling and discussing best practices. Each module integrates best practices taken from the most current research including modeling, language supports, connections to curricula, and connections to big ideas.

Educators take this course with a cohort of their peers who want to learn effective standards-aligned and evidence-based practices for teacher about fractions with all learners. Participants are expected to have regular access to computers and proficiency with email and current internet browsers. The course facilitator will answer questions, monitor work and progress, provide feedback, and monitor and add onto course discussion boards.

Learning Outcomes

At the conclusion of this course, participants will be able to:

1. Understand the progression of fraction concepts through multiple grade levels.
2. Recognize the connections between whole number and fraction concepts and procedures.
3. Recognize the connections within fraction concepts and procedures.
4. Identify, connect, and understand how to use multiple representations of fractions (concrete, visual/representational and abstract) to teach fraction concepts and operations.
    - Meaning of fractions
    - Comparing fractions
    - Operations with fractions (addition and subtraction)

5. Access physical, digital, and visual models of fractions as means to make fraction concepts clear for students.
6. Understand strategies to address common misconceptions with fractions.
7. Connect content standards and the Standards for Mathematical Practices to the teaching and learning of fractions.
8. Explain the recommendations and rationale from research for best practices in teaching about fractions.
9. Apply learning to their own teaching (or coaching).

Teaching/Learning Strategies

- Working through and studying online explanation and examples.
- Watching video clips utilizing online resources to illustrate examples for teaching.
- Completing interactive learning activities with built in feedback.
- Completing weekly Journal reflections.
- Completing self-check "quizzes" with immediate feedback.
- Participating in weekly online discussions with peers.
- Developing further learning through independent assignments of choice.
- Conferencing with the course instructor.

Optional Reading

Elementary and Middle School Mathematics Teaching Developmentally (Van de Wall, Karp and Bay-Williams, any edition from 2007 - 2019 will be sufficient, 6th edition through 10th edition). Within each homework is a reading **option**. If participants choose to do the reading option of the assigned homework with each module, they will need access to the Van de Wall, Karp and Bay-Williams book. The price of the book is not included in the cost of the course since work in the book is optional. The book can be rented on Amazon or directly from Savvas for approximately $40 or purchased for about $80.

College Credit Option

Participants may receive 1 graduate-level semester extension unit for the course for an additional cost of $109 per. These units are typically used for salary advancement. There are no additional requirements beyond successful completion of this 16-hour course.

See Course Outline on the following pages.

Fraction Course 1 – Fraction Meaning, Equivalence, Addition and Subtraction

Week 1:
- Module 1: Orientation and Introduction
- Module 2: Meaning and Modeling of Fractions.

Week 2:
- Module 3: Equivalence
- Module 4: Comparing Fractions

Week 3:
- Module 5: Adding Fractions

Week 4:
- Module 6: Subtraction Fractions
- Module 7: Course Wrap Up

Module 1: Course Orientation and Introduction	
The course orientation is designed to help you get to know (or review) some basics about online learning in general and this course in particular. You will learn about the layout of the course, how to navigate the course, and learn about and experience several types of activities that are used throughout the course. The course introduction is designed to orient you to the course, review recommendations from research for the teaching and learning of fraction concepts and procedures.	
Sections in Module to Read and Study	**Related Activities to Complete**
Complete the *Orientation* section. • Orientation Intro • Tips for Online Learning • About this Course • Types of Pages and Assignments • Packet: Types of Pages in the Course	• Video – Course Introduction • Short Answer – Question about a Book • Multiple Choice - Homework Preference • Matching – Identify Course Icons • Discussion Forum: Introduce Yourself • Orientation Survey
Complete the *Introduction* section. • Welcome • Course Objectives • Design of the Course • Rationale for the Course	• Video – Course Introduction • Anticipation Guide • Short Answer - “What can you say about 6/8?”
Complete the *Practices and Proficiencies* section. • Five Proficiency Strands in Mathematics • Standards for Mathematical Practice • Critical Topics in Fractions	• Matching - Proficiency Strands with Instructional Episodes • Journal – Share learning from Dana Center Videos on Math Practices • Journal – Identify fraction progression and key vocabulary in your own state standards

Module 2: Meaning and Modeling

With Meaning and Modeling we begin exploring, learning about, and discussing the meaning, representation of and uses of fractions.

The big mathematical ideas (Van de Walle, Karp, and Bay-Williams 2019) that are the focus of Module 1 are:

- Fractions can and should be represented across different interpretations (e.g., part-whole and division) and different models: area (e.g., 1/3 of a garden)), length (e.g., ¾ of an inch), and set (e.g., ½ of the marbles).
- Fractions are equal shares of the a whole or a unit. Therefore, equal sharing activities (e.g., 2 sandwiches shared with 4 friends) build on whole-number knowledge to introduce fractional quantities.
- Partitioning and iterating are strategies students can use to understand the meaning of fractions. Partitioning can be thought of as splitting the whole equally (e.g., spitting a whole into fourths), and iterating can be thought of as making a copy of each piece and counting them (e.g., one-fourth, two-fourths, etc.).

(p. 337)

Sections in Module to Read and Study	**Related Activities to Complete**
Complete the *Foundational Understandings* section. • Meaning of Fractions • Unitizing • Paper Folding	• Short Answer – Compare 1/2 and 1/3 • Videos – Modeling Paper Folding • Journal – Module reflection **Homework:** A. Option 1: Reading from Elementary and Middle School Mathematics and Journal reflection. B. Option 2: Analyze Your Curriculum to see how fractions are initially made sense of and which models are used to represent fractions.

Module 3: Equivalent Fractions and Comparing Fractions

In this module you will

- Use visual models that can flexibly represent equivalent fractions.
- Connect visual models to abstract methods for creating equivalent fractions.

The big mathematical ideas (Van de Walle, Karp, and Bay-Williams 2019) that are the focus of Module 1 are:

> ➢ Equivalent fractions are ways of describing the same amount by using different-sized fractional parts.
>
> (p. 337)

Sections in Module to Read and Study	Related Activities to Complete
Complete the *Introduction* section. • Objectives and Big Ideas • Initial Equivalent and Fraction Comparison Discussion	• Discussion Forum – Challenges for Students with Equivalence or Comparing
Complete the *NAEP Problems* section. • NAEP Problem "Closest to 1/2" • NAEP Problem "Order from Least to Greatest"	• Multiple Choice – Closest to ½ + Short Answer– Closest to ½ • Multiple Choice – Order from Least to Greatest +Short Answer – Order from Least to Greatest
Complete the *Multiplicative Identity* section. • Connect Equivalence to Whole Numbers • Comparing Like Units • The Multiplicative Identity	• Video – The Multiplicative Identity • Multiple Choice – Check Your Understanding
Complete the *Connecting Representations* section. • Equivalent Fractions with Area Models • Equivalent Fractions on the Number Line • Modeling with NCTM Illuminations • Fractions Greater Than One	• Journal – Using Visual Models • Video – Demo NCTM Activity

Module 4: Comparing Fractions

In this module you will

- Apply multiple techniques for comparing fractions.
- Engage in a variety of activities for comparing and ordering fractions.
- Share and discuss ideas related to teaching about equivalent fractions and comparing fractions.
- Extend knowledge through classroom application or further study.

The big mathematical idea (Van de Walle, Karp, and Bay-Williams 2019) that are the focus of Module 1 are:

> ➢ Fractions can be compared by reasoning about the relative size of the fractions. Estimation and reasoning are important in teaching understanding of fractions.
>
> (p. 337)

Sections in Module to Read and Study	Related Activities to Complete
Complete the *Comparing Fractions* section. • Paper Folding • Comparing Fractions Using Area Models • Activity Matching Area Models with Fraction Pairs • Comparing Fractions Using Tape Diagrams and Circles • Comparing Fractions Using the Number Line • Fractions on the Number Line Activity • More Fractions on the Number Line • Comparing Fractions by Connecting Visuals with Free Applications • Comparing Fractions on a Clothesline Number Line • Compare Pairs of Fractions Using Reasoning About Fraction Concepts • Compare Three Fractions Using Reasoning About Fraction Concepts • Visual Comparison of Three Fractions	• Short Answer – Connecting Paper Folding • Matching – Area Models and Fraction Pairs • Matching – Fractions on a Number Line • Video – Math Learning Center demo • Video – Clothesline Math with Fractions • Journal – reflection on exploring online resources • Matching & Essay – Comparing Pairs of Fractions • Essay – Compare Three Fractions
Complete the *Fraction Activities, Wrap Up and Homework* sections • Counting Up and Down Fraction Activity • Fraction War • Wrap Up • Homework	• Video – Modeling Counting Up and Down • Journal – Module reflection **Homework:** A. Option A: Apply learning to classroom instruction and reflect on outcomes. B. Option B: Read/share from *Elementary and Middle School Mathematics*. C. Option C: Analyze Your Curriculum to connect to ideas from this module.

Module 5: Adding Fractions

In this module, you will:

- Use visual models that can flexibly represent addition with fractions.
- Connect visual models to number line and numerical methods for adding fractions.
- Estimate sums with fractions.
- Learn best practices for teaching procedures with fractions.
- Engage in a variety of activities for adding fractions.
- Share and discuss ideas related to teaching fraction addition and subtraction.
- Extend knowledge through classroom application or further study

The big mathematical ideas (Van de Walle, Karp, and Bay-Williams 2019) that are the focus of Module 3 are:

- The meanings of each operation with fractions are the same as the meanings for the operations with whole numbers. Operations with fractions should begin by applying these same meanings to fractional parts.
- For addition and subtraction, the numerator tells the number of parts and the denominator the unit. The parts are added or subtracted.
- Estimation should be an integral part of computation development to keep students' attention on the meanings of the operations and the expected sizes of the results.

(pp. 373-74)

Sections in Module to Read and Study	Related Activities to Complete
Complete the *Introduction* section. • Objectives and Big Ideas • Sharing Perspectives on Fractions • Recommendations for Effective Teaching Practices • Estimation • Problem Types	• Discussion Forum – Reflecting on Perspectives on Teaching Fractions • Journal – Reflecting on Effective Practices • Short Answer – Estimate and Explain
Complete the *Addition* section. • Addition and Subtraction Overview • Modeling Addition with Online Visual Tools • Modeling Addition on a Number Line • Modeling Addition with Paper and Pencil • Estimation with Addition	• Video – Modeling Visual Models for Fraction Addition • Matching – Visual Models and Expressions for Addition • Video – Modeling Addition on a Number Line • Videos – Modeling Addition with Paper and Pencil • Short Answer – Estimation Challenge for Fraction Addition

Module 6: Subtracting Fractions

In this module, you will:

- Use visual models that can flexibly represent subtraction with fractions.
- Connect visual models to number line and numerical methods for subtracting fractions.
- Estimate sums and differences with fractions.
- Learn best practices for teaching procedures with fractions.
- Engage in a variety of activities for adding and subtracting fractions.
- Share and discuss ideas related to teaching fraction addition and subtraction.
- Extend knowledge through classroom application or further study

The big mathematical ideas (Van de Walle, Karp, and Bay-Williams 2019) that are the focus of Module 3 are:

- The meanings of each operation with fractions are the same as the meanings for the operations with whole numbers. Operations with fractions should begin by applying these same meanings to fractional parts.
- For addition and subtraction, the numerator tells the number of parts and the denominator the unit. The parts are added or subtracted.
- Estimation should be an integral part of computation development to keep students' attention on the meanings of the operations and the expected sizes of the results.

(pp. 373-74)

Sections in Module to Read and Study	Related Activities to Complete
Complete the *Subtraction* section. • Modeling Subtraction with Online Visual Tools • Modeling Subtraction on a Number Line • Modeling Subtraction with Paper and Pencil • Estimation with Subtraction • Reflection on Fraction Models	• Video – Modeling Connecting Visual Models to Numerical Methods for Subtracting Fractions • Matching – Visual Models and Expressions for Subtraction • Video – Modeling Subtraction on a Number Line • Videos – Modeling Subtraction with Paper and Pencil • Short Answer – Estimation Challenge for Fraction Subtraction • Discussion Forum: Reflecting on the Use of a Variety of Visual Models
Complete the *Mixed Numbers* section • Addition with Mixed Numbers • Subtraction with Mixed Numbers	• Short Answer – Mental Addition Challenge • Videos – Connecting Visual and Numerical Methods for Mixed Number Subtraction
Complete the *Misconceptions and Problem Types* section • Common Misconceptions • Problem Types Revisited	• Discussion Forum – What Misconceptions Do You Commonly See? • Matching – Problem Types

Complete the *Challenge Problems* section • Challenge Problems	• Fill In – Greatest and Least Sums and Differences • Fill In – Spend Some Time with 1 to 9 with Unit Fractions • Short Answer – Serena Paints Her Fence
Complete *the Wrap Up and Homework* section • Wrap Up • Homework	• Journal – Module reflection **Homework:** A. Option A: Apply learning to classroom instruction and reflect on outcomes. B. Option B: Read/share from *Elementary and Middle School Mathematics*. C. Option C: Analyze Your Curriculum to connect to ideas from this module.

Module 7: Course Wrap Up– Reflection and Exploration	
In the final module of the course, you will: • Revisit the goals and objectives of the course. • Consider how new learning from this course will shift your practice. • Review and reflect on the methods for developing the Standards for Mathematical Practice in students. • Explore additional online resources for fraction activities.	
Sections in Module to Read and Study	**Related Activities to Complete**
Complete the *Reflecting* section. • Revisiting the Goals and Objectives • Progression of Critical Standards • Importance of the Standards for Mathematical Practice	• Video – Course Summary • Journal – Revisiting the Anticipation Guide • Journal – Describing Key Take-aways • Journal – Connections to SMPs • Video – Farewell Message • Explore – Explore Additional Online Resources for Work with Fractions

Course Resources and References

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