CORE MATH®

Five Favorite Strategies for Teaching About Fractions

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Today's Presenter

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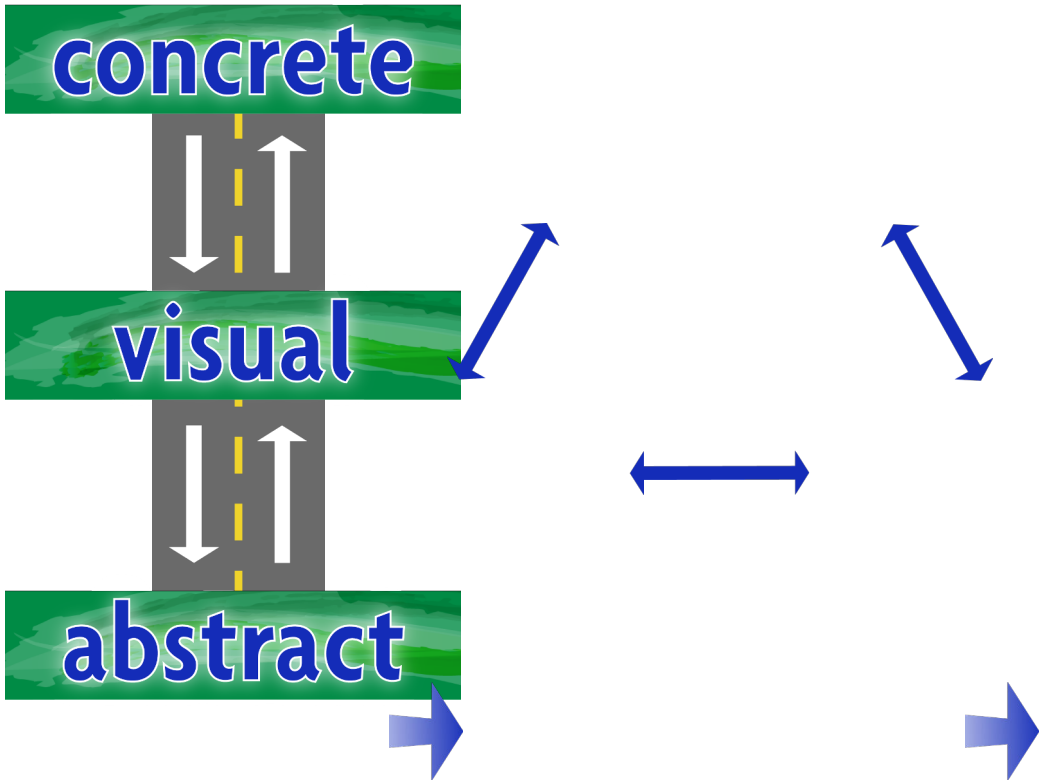
Objectives

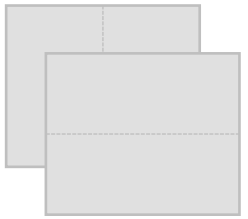
- See physical and visual models that are flexible, doable, and clearly connect fraction concepts.
- Recognize connections between fraction concepts and whole number concepts.
- Learn how fraction concepts build on each other in sensible ways.
- Experience challenge problems with fractions that extend and assess student understanding.
- Gain ideas for fluency building activities that are fun and effective.

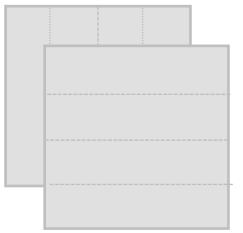
Progression of Learning
Concrete – Visual - Abstract

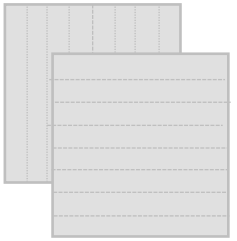


Paper Folding 1

- Students work in pairs. Each student has three pieces of letter size paper (8½" by 11").
- Fold one paper in half. What does this make? Record.
  - One student fold vertically, and one fold horizontally
- Fold another paper in half twice. What does this make?
  - One student fold vertically, and one fold horizontally
- Fold another paper in half 3 times. What does this make?
  - One student fold vertically, and one fold horizontally
- Compare / discuss, how many fourths make a half? How many eighths make a half? How many eighths make a fourth? Are you and your partner's halves equal? Why?

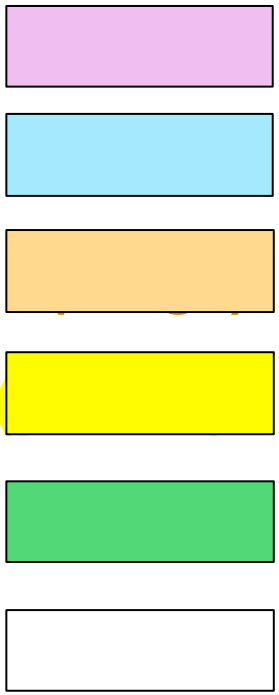






Visualize Fractions with Paper Folding

1. Teach using visual representations. Write the fraction on these strips in words on one side, in symbols on the other side.
   - Fold the pink paper strip into halves
   - Fold the blue paper strip into fourths
   - Fold the beige paper strip into eighths
   - Fold the yellow paper strip into thirds
   - Fold the green paper strip into sixths
   - Fold the white paper strip into fifths

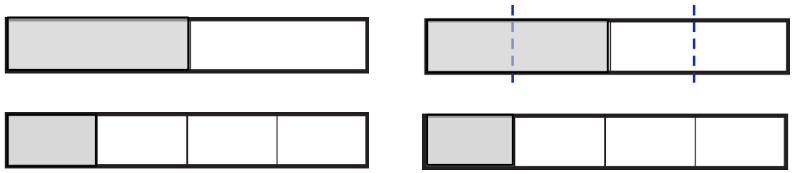


2. What does paper folding teach about fractions?

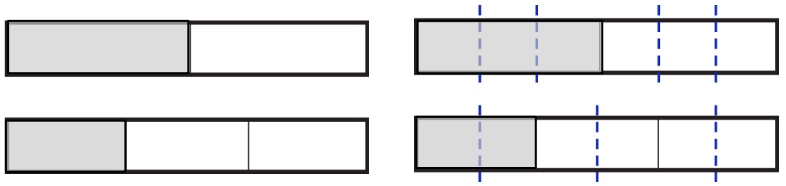
Comparing Fractions with - Tape Diagrams

How can I change each tape diagram so that both have the same-size parts?

$\frac{1}{2}$ and $\frac{1}{4}$



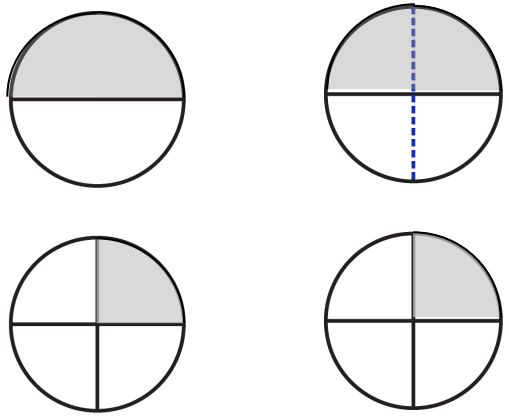
$\frac{1}{2}$ and $\frac{1}{3}$



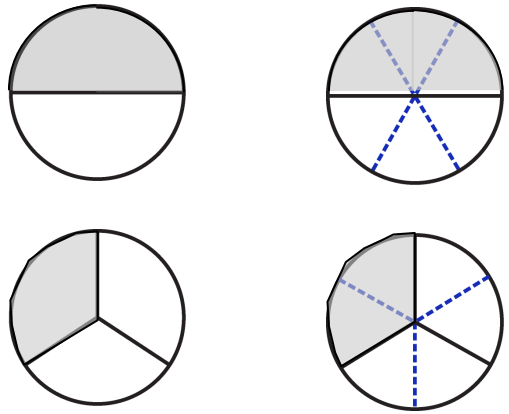
Circle Diagrams

How can I change each circle diagram so that both have the same-size parts or the same **UNITS**?

$\frac{1}{2}$ and $\frac{1}{4}$



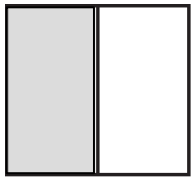
$\frac{1}{2}$ and $\frac{1}{3}$



Using Area Models to Visualize Multiplication

¼ x ½

¼ of ½



$1/4 \times 1/2 =$
$1/4 \times 2 = 1/8$

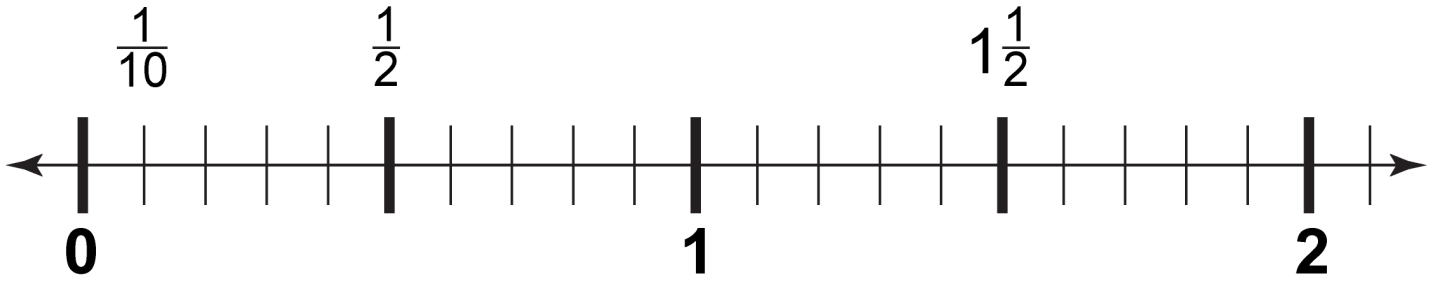
$1/2 \times 1/4 =$
$1/2 \times 4 = 1/8$

$1/3 \times 1/2 =$
$1/3 \times 2 = 1/6$

$1/2 \times 1/3 =$
$1/2 \times 3 = 1/6$

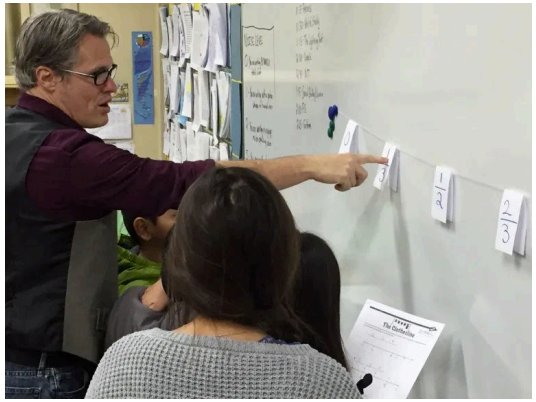
Number Lines - Fractions as Numbers

A fraction is a representation of a number. As such it can be placed on the number line.

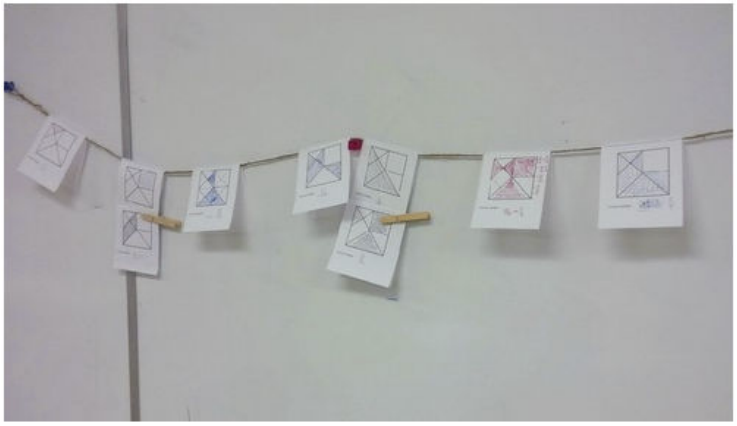


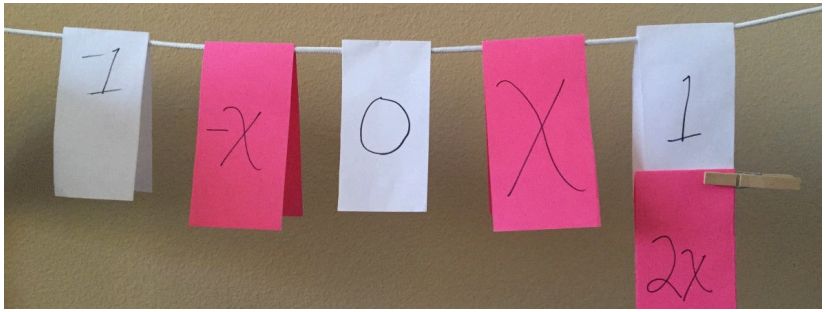
Clothes Line Number Lines with Fractions

Chase Orton
UndercoverCalculus.com



FractionTalks.com

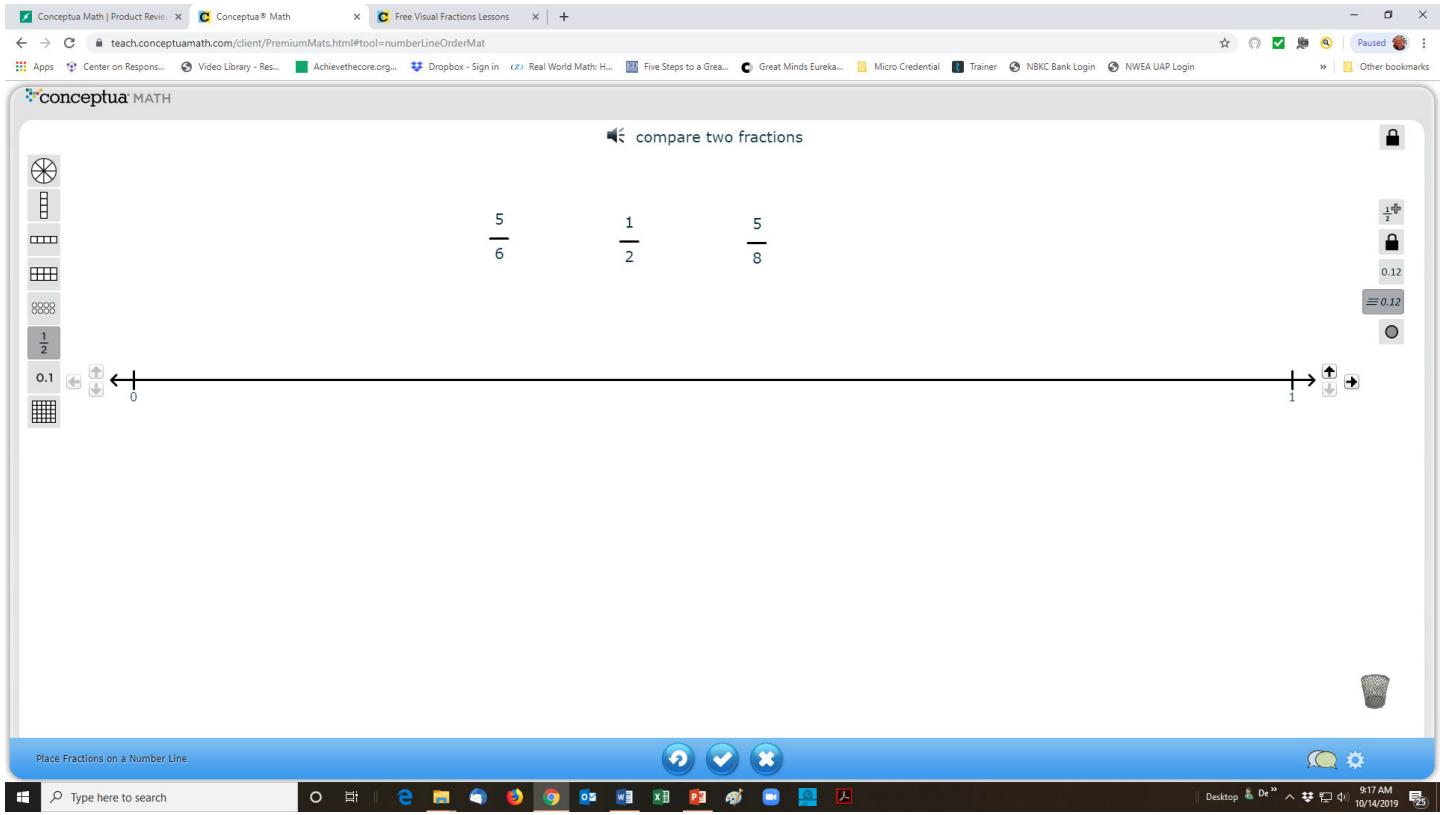




ClothesLineMath.com

Online Visual Tools

- ConceptuaMath.com



Wrap Up – Concrete – Visual - Abstract

Importance of using concrete and visual models connected with and leading to abstract or symbolic numerical representations of fractions, fraction properties and operations with fractions.

- Paper folding activities
- Rectangular diagrams/tape diagrams
- Number lines (& clothes line number lines)
- Online visual tools (such as ConceptuaMath)

Connecting Fractions with Whole Numbers

- Concept of Units
- Fractions as Numbers
- Equivalent Fractions
- Addition and Subtraction
- Multiplication and Division

Directions in Standards for Connections

Grade 3	Grade 4	Grade 5	Grade 6
• Unit fractions • Part-whole • Equal parts • Same size wholes • Fractions as numbers • Compare fractions • Use visual models	• Equivalent fractions • Use unit fractions to compose and decompose fractions • Use previous understandings with operations to understand addition, subtraction, and multiplication of fractions • Use visual models • Solve word problems for addition and subtraction • Decimal fractions	• Apply understanding of fractions to add and subtract with unlike denominators • Fluency with addition and subtraction • Estimate sums and differences • Use previous understandings with operations to understand multiplication and division • Make sense of multiplication and division • Solve word problems for addition, subtraction multiplication, and division • Use visual models	• Compute quotients with fractions. • Interpret quotients • Solve word problems with fraction operations • Use visual models • Use equations

UNITS – From Place Value to Fractions

Place Value

thousands, hundreds, tens,

ONES,

tenths, hundredths, thousandths

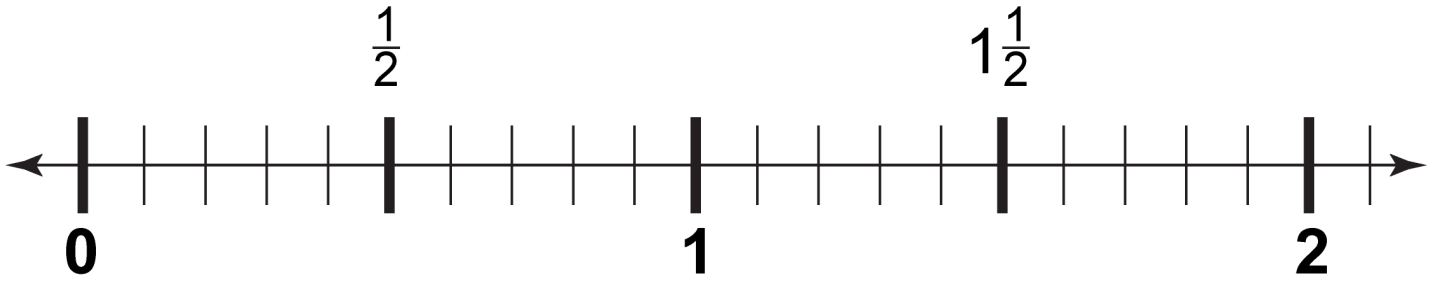
Units

- Units – ones, tens, hundreds, thousands . . .
- Units – tenths, hundredths, thousandths . . .
- Units – halves, thirds, fourths, fifths, tenths …

Denominator - describes the **units** with fractions, based on partitioning the "ones" unit.

Number Lines - Fractions as Numbers

A fraction is a representation of a number. As such it can be placed on the number line.



Equivalent Fractions

Multiplicative identity property:

Any number x 1 = the same number

18 x 1 = 18

Same property is central to creating equivalent fractions.

$$\frac{2}{5} \times 1 = \frac{2}{5}$$

However, the end result doesn't always look the same.

Suppose we multiply 2/5 by 3/3? The results looks like a different value.

$$\frac{2}{5} \times \boxed{1}\ \frac{3}{3} = \frac{6}{15}$$

Use prior visual models, and talk about multiplying by the "big bad one."

Multiple Equivalent Representations

Which of these is equivalent to 451?

a) 4 hundreds + 5 tens + 1 one
b) 4 hundreds + 4 tens + 11 one
c) 3 hundreds + 15 tens + 1 one
d) 3 hundreds + 14 tens + 11 ones

$$\begin{array}{r} 451 \\ -\ 273 \\ \hline \end{array}$$

Which of these is equivalent to 2/5?

a) 2 fifths
b) 4/10
c) 8/20
d) 6/15

$$\frac{2}{5} - \frac{2}{15} = \frac{6}{15} - \frac{2}{15}$$

Addition and Subtraction – Like Units

Whole numbers:

Combine like units

- Ones with ones
- Tens with tens
- Hundreds with hundreds

$$\begin{array}{r} 451 \\ +\ 243 \\ \hline 694 \end{array}$$

Fractions:

Combine like units (denominators are the units!)

- Fourths with fourths,
- Fifths with fifths,
- Fifteenths with fifteenths . . .

$$\frac{2}{5} - \frac{2}{15} = \frac{6}{15} - \frac{2}{15} = \frac{4}{15}$$

Emphasize and Name the Units

Whole numbers:

40 + 50 = 90 → 4 tens + 5 tens = 9 tens

Fractions:

1/5 + 3/5 = 4/5 → 1 fifth + 3 fifths = 4 fifths

Multiplication

Multiplication of Whole Numbers:

$3 \times 5 = 5 + 5 + 5$

Multiplication of Fractions:

$3 \times 1/5 = 1/5 + 1/5 + 1/5$

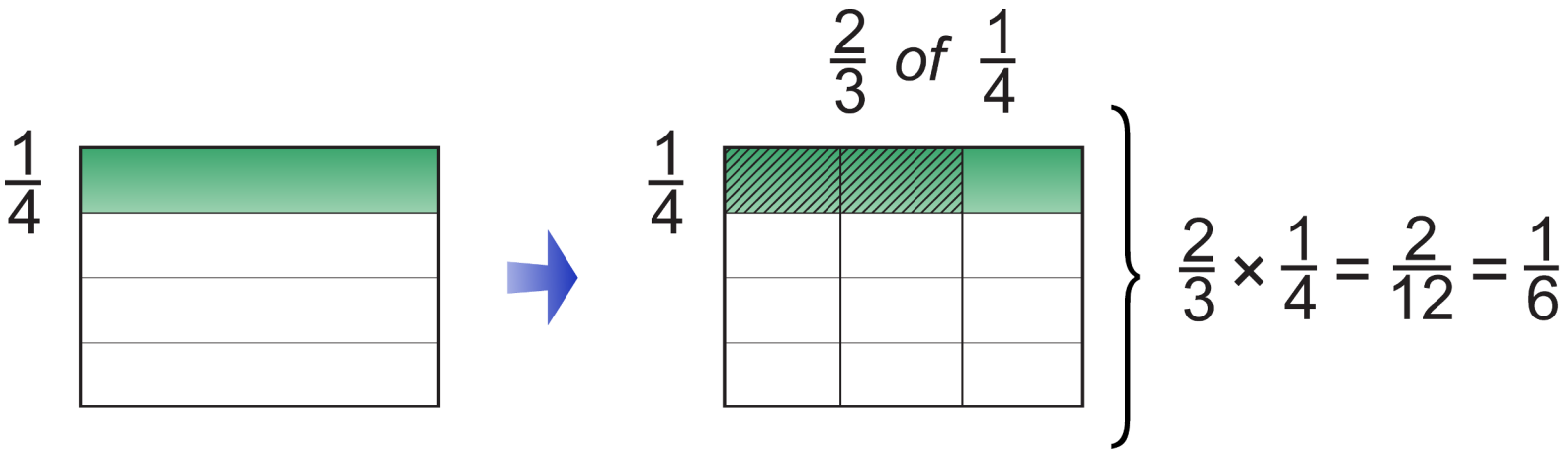
Multiplication of Fractions:

$3 \times 2/5 = 2/5 + 2/5 + 2/5$

$= (3 \times 2) / 5 = 6/5$

Fraction Multiplication with an Area Model

$$\frac{2}{3} \times \frac{1}{4}$$



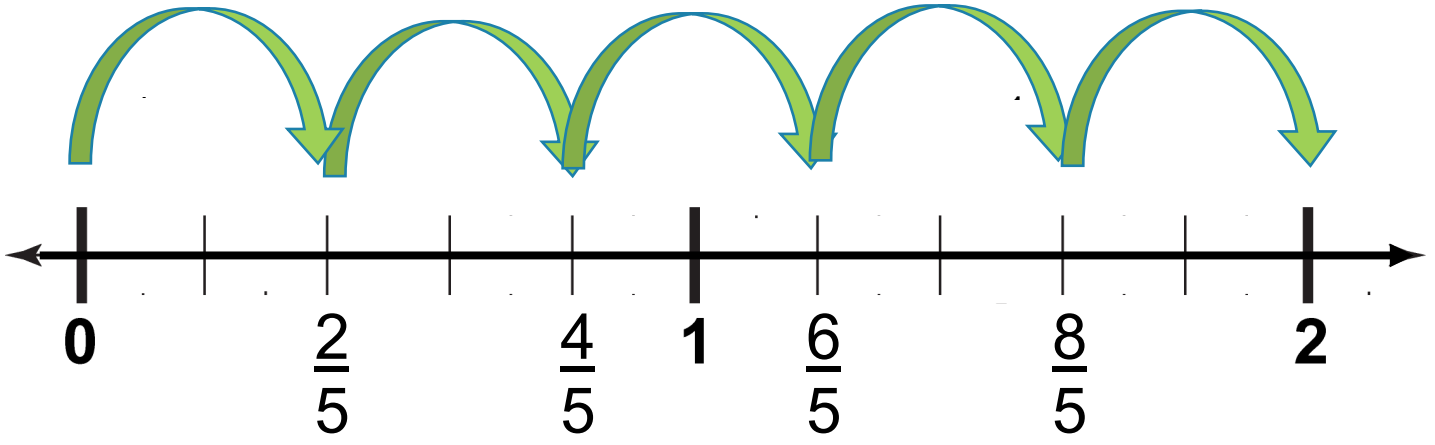
Division – How Many In the Group?

Division with Whole Numbers:
12 divided by 4 → How many fours are in 12?

- There are three 4s in 12. 12 divided by 4 = 3.

Division with Fractions:
2 Divided by 2/5 → How many 2/5^{ths} are in 2?

- **There are five 2/5^{ths} in 2. Therefore 2 ÷ 2/5 = 5.**

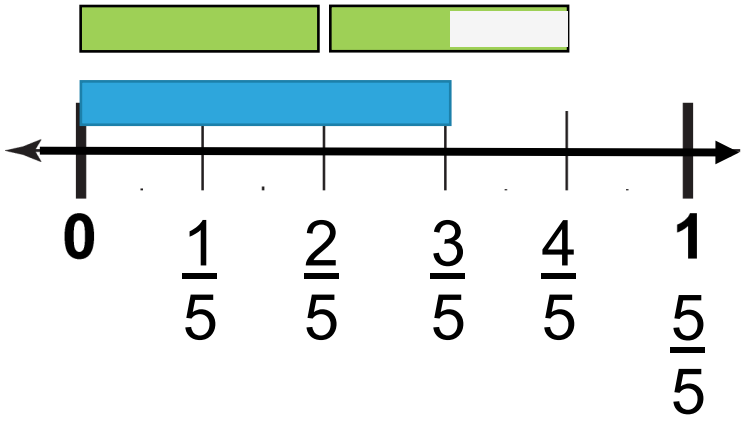


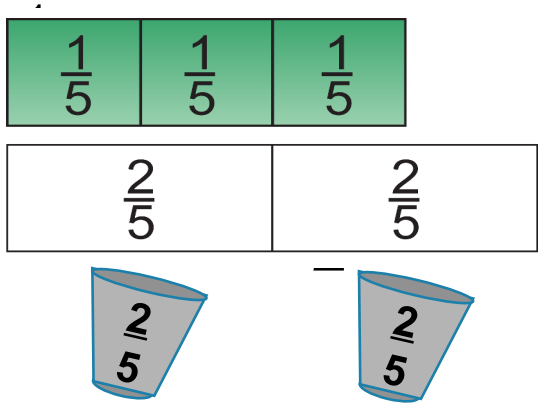
Batches of Muffins

I have $\frac{3}{5}$ cup of sugar left in my sugar container. I need $\frac{2}{5}$ of a cup of sugar for each whole batch of muffins. How many batches of muffins can I make?

$$\frac{3}{5} \div \frac{2}{5} = \text{number of batches of muffins}$$

There are 1½ 2/5^{ths} in 3/5^{ths} . Therefore 3/5 ÷ 2/5 = 1½.



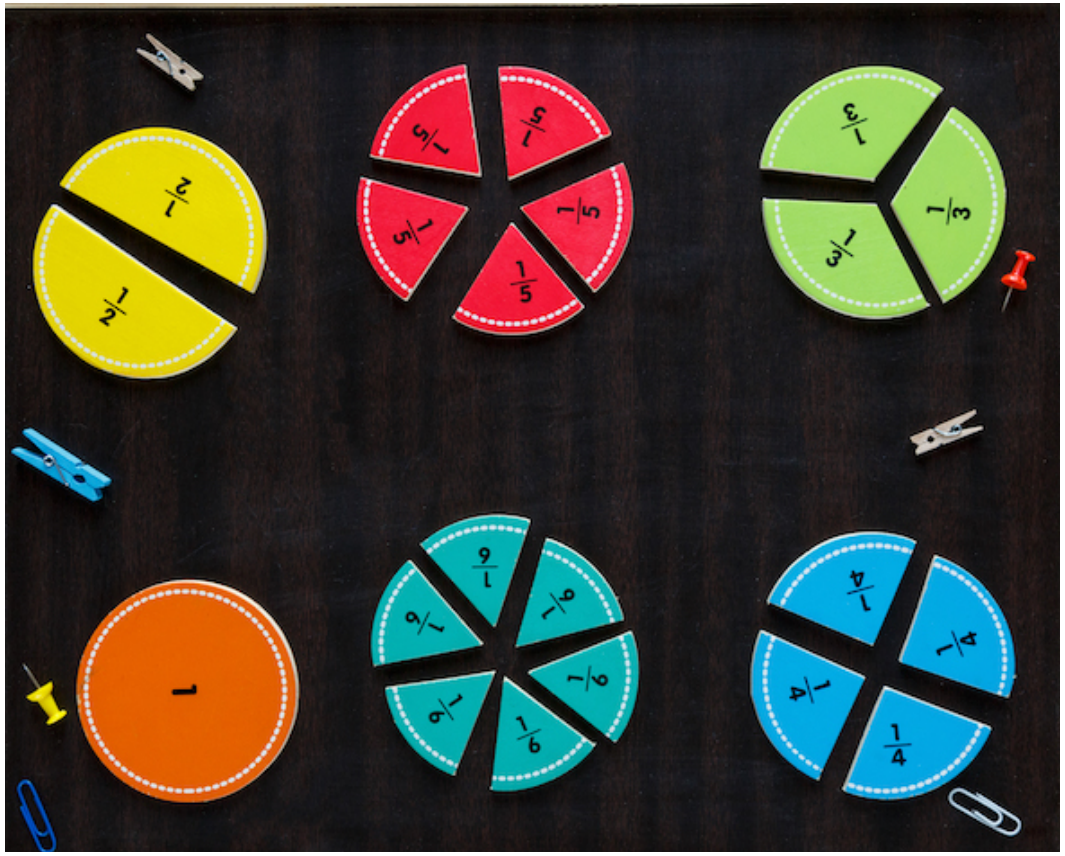


Wrap Up – Connecting Fractions with Whole Numbers

- **Concept of Units** – the denominator is the unit based on some part of the ”ones” unit.
- **Fractions as Numbers** – fit on a number line just like whole numbers and can be used to count parts of objects.
- **Equivalent Fractions** – Big Idea that numbers can be represented in many equivalent forms and we use different versions based on need.
- **Addition and Subtraction** – Combine like units
- **Multiplication and Division** - Repeated addition, area model, how many of one quantity is in the other quantity.

Connections Among Fraction Concepts

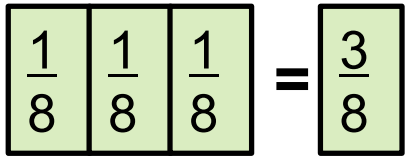
- Building fractions from unit fractions
- Patterns in division of fractions



Building Fractions from Unit Fractions

Use unit fractions to compose and decompose fractions. (CCSSM 2010)

- With whole numbers we build on the "ones" unit.
- With fractions we build other fractions from unit fractions (iteration).



$$3/8 = 3(1/8) = 1/8 + 1/8 + 1/8 = 2/8 + 1/8$$

$$11/8 = 8/8 + 3/8 = 1 + 3/8 = 13/8$$

Patterns with Fraction Division

- Fill in the tables with the correct quotients.
- Describe any patterns you notice.
- Work with a partner to try to come up with a shortcut for dividing a whole number by a fraction. For example, what is 5 divided by 3/8?

dividend

divisor

Expression	Quotient
$3 \div 3$	
$3 \div 1$	
$3 \div \frac{1}{2}$	
$3 \div \frac{1}{3}$	
$3 \div \frac{1}{4}$	
$3 \div \frac{1}{5}$	
$4 \div \frac{1}{4}$	

Expression	Quotient
$3 \div \frac{1}{4}$	
$3 \div \frac{2}{4}$	
$3 \div \frac{3}{4}$	
$6 \div \frac{1}{4}$	
$6 \div \frac{2}{4}$	
$6 \div \frac{3}{4}$	

Expression	Quotient
$3 \div \frac{1}{4}$	
$\frac{3}{2} \div \frac{1}{4}$	
$\frac{3}{4} \div \frac{1}{4}$	
$\frac{3}{12} \div \frac{1}{4}$	

What is 4/5 divided by 1/8?

Learning About Fractions

- Concrete – Visual – Abstract
- Connecting fractions to whole numbers concepts
- Connecting fractions to other fraction concepts
  - Building on unit fractions
  - The division algorithm through patterns

Closest to $\frac{1}{2}$

Which fraction has a value closest to $\frac{1}{2}$?

A. $\frac{5}{8}$ B. $\frac{1}{6}$ C. $\frac{2}{2}$ D. $\frac{1}{5}$

Which is the most popular incorrect answer?

4th-Grade NAEP, 2009

25% answered correctly (A)

40% chose C

Thinking about Division

6-8: Examine the four division problems shown below.

Without calculating the quotients, which quotient is closest to 1? Explain and/or show your reasoning.

A. $19/20 \div 1/18$

B. $1/20 \div 1/18$

C. $1/4 \div 4$

D. $4 \div 1/4$

A number divided by itself equals one (except 0).

Comparing Fractions Without Using Common Denominators (or decimals or percent)

Compare the following pairs of fractions **without** converting to common denominators, common numerators, decimals, or percents, or using a number line. Explain your reasoning.

1. 3/ < d 5/8

3/7 *is less than half*

5/8 *is greater than half*

2. 5/ > d 5/8

Sixths are greater than eighths (same size wholes)

3. 5/ > d 3/4

5/6 *is* 1/6 *from* 1

3/4 *is* 1/4 *from* 1

Spend Some Time with 1 to 9

Challenge 18 **Spend Some Time with 1 to 9, K–8**

Spend a Fraction of Equal Time with 1 to 9

Fill in each box with a digit from 1 to 9 so that equivalent fractions are created.

- No digit may be repeated in the same set of equivalent fractions.

For example:

This is correct because the three fractions are equal and no digit is used more than once.

$$\frac{1}{2} = \frac{5}{3+7} = \frac{4}{8}$$

This is not correct because the digit 2 is used more than once in the set of three fractions.

$$\frac{1}{2} = \frac{5}{8+2} = \frac{3}{6}$$

1. $\frac{1}{3} = \frac{\square}{\square} = \frac{\square}{\square + \square}$

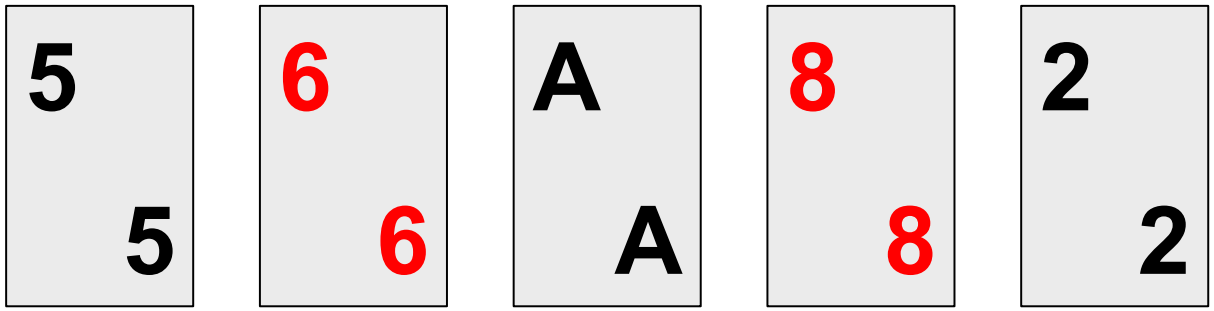
2. $\frac{1}{4} = \frac{\square}{\square} = \frac{\square}{\square + \square}$

Develop Fluency Through Engaging Activities

- Card Games
- Counting Activities

Card Game – Fraction War

- Standard deck of cards
- Divide cards up between two players
- Each player mixes his/her cards face down in a stack.
- Each player turns over her/his first two cards.
- Each player uses their own two cards to create a fraction equal to or less than one.
- The player with the greater fraction wins the round.
- Optional: Record fractions on a play sheet.



Counting Up and Down with Fractions

- Count by halves starting at 1
- Count by halves starting at 4½.
- Count by halves starting at ¼ (1¾, 2¼, 2¾, 3¼, 3¾, 4¼, 4¾)

Advice:

- Start small with very doable numbers
- Use very clear hand signals for counting up and down
- Focus on where students are at to move forward
- Require students to stay with your hand signals
- Go back and forth across whole numbers
- **Discuss patterns and challenges**
- **Use a number line to visualize & build understanding**

Ideas We Explored Today

- Concrete – Visual - Abstract
- Connect fraction concepts with whole number concepts.
- Build fraction concepts on other fraction concepts
- Challenge problems to make us think.
- Engaging activities to build fluency

Antelope Canyon, Navajo Reservation

Q & A and Let's Connect!

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