CORE MATH®

4 Must Do's for Math Instruction

Dean Ballard

Director of Mathematics, CORE Inc.

www.corelearn.com

Today's Presenter

Dean Ballard
Director of Mathematics
CORE

Essential Questions to Explore Today

- How do explicit instructional techniques apply to both direct and inquiry-based instruction?
- Why is student discourse important and what are tips for generating meaningful discourse?
- Why is vocabulary important in math and what are strategies for building math vocabulary?
- What are some easy to use differentiation techniques and how these help students?

Antelope Canyon, Navajo Reservation

What is it, exactly, about teaching that influences students' learning of mathematics?

- *Teaching that attends explicitly to concepts – to connections among mathematical facts, procedures, and ideas.*
- *The engagement of students in struggling or wrestling with important mathematical ideas . . . that students expend effort to make sense of mathematics, to figure something out that is not immediately apparent . . . solving problems that are within reach.*

Hiebert and Grouws 2007

Math Jokes ☺

1. What type of gin do mathematicians prefer in mixed drinks? **ORI-GIN**

2. What do little acorns say when they realize they are grown up? **GEOMETRY (Gee, I'm a tree)**

3. What should one do when it rains?
   COINCIDE (Go inside)

4. What animal is known as the king of the beasts? **LINE (Lion)**

5. The morning after the angle had too much to drink, what did it feel like?
   RECTANGLE (wrecked angle)

Rectangle
Origin

Coincide
Geometry
Algebra

What Is Explicit Instruction?

"Explicit instruction is a systematic instructional approach that includes a set of delivery and design procedures derived from effective schools research . . ."

Ideas That Work

". . . unambiguous and direct approach to teaching that incorporates instruction design and delivery."

Archer & Hughes 2011

Elements of Explicit Instruction

Design:

1. Instruction focuses on critical content.
2. Break down complex skills and strategies.
3. Provide systematic instruction:
    - Organized and focused lessons
    - Clear statement of goals and expectations
    - Review of prior skills and knowledge
    - Step-by-step demonstrations
4. Provide judicious practice

Elements of Explicit Instruction (cont.)

Delivery:

5. Require frequent responses.
6. Monitor student performance closely.
7. Provide immediate affirmation and corrective feedback.
8. Deliver the lesson at a brisk pace.
9. Help students organize knowledge.

Adapted from Archer & Hughes 2011

Explicit, Direct, and Inquiry-based Instruction

- ***Explicit Instruction*** – A set of instructional techniques that work effectively together. These should be applied to both direct instruction and inquiry-based instruction.
  - ***Direct Instruction*** – Teacher telling and modeling to students what she/he wants students to know and do.
  - ***Inquiry-based Instruction*** – Students explore an idea and reach toward or arrive at some important conclusion(s).

Not Either/Or

All-encompassing recommendations that instruction should be entirely "student centered" or "teacher directed" are not supported by research.

National Mathematics Advisory Panel 2008

Effect Sizes from Hattie in *Visible Learning for Mathematics*

- Direct instruction – 0.59
- Dialogic instruction – 0.82

This doesn't mean teachers should choose one approach over another. It should never be an either/or situation. . . . Both dialogic and direct instruction have a role to play throughout the learning process, but in different ways. (p. 24)

Hattie, Fisher, and Frey 2017

Direct Instruction

The teacher decides the learning intentions and success criteria, makes them transparent to the students, demonstrates them by modeling, evaluates if they understand what they have been told by checking for understanding, and re-telling them what they have told by tying it all together with closure (p. 206).

Hattie in *Visible Learning* (2009)

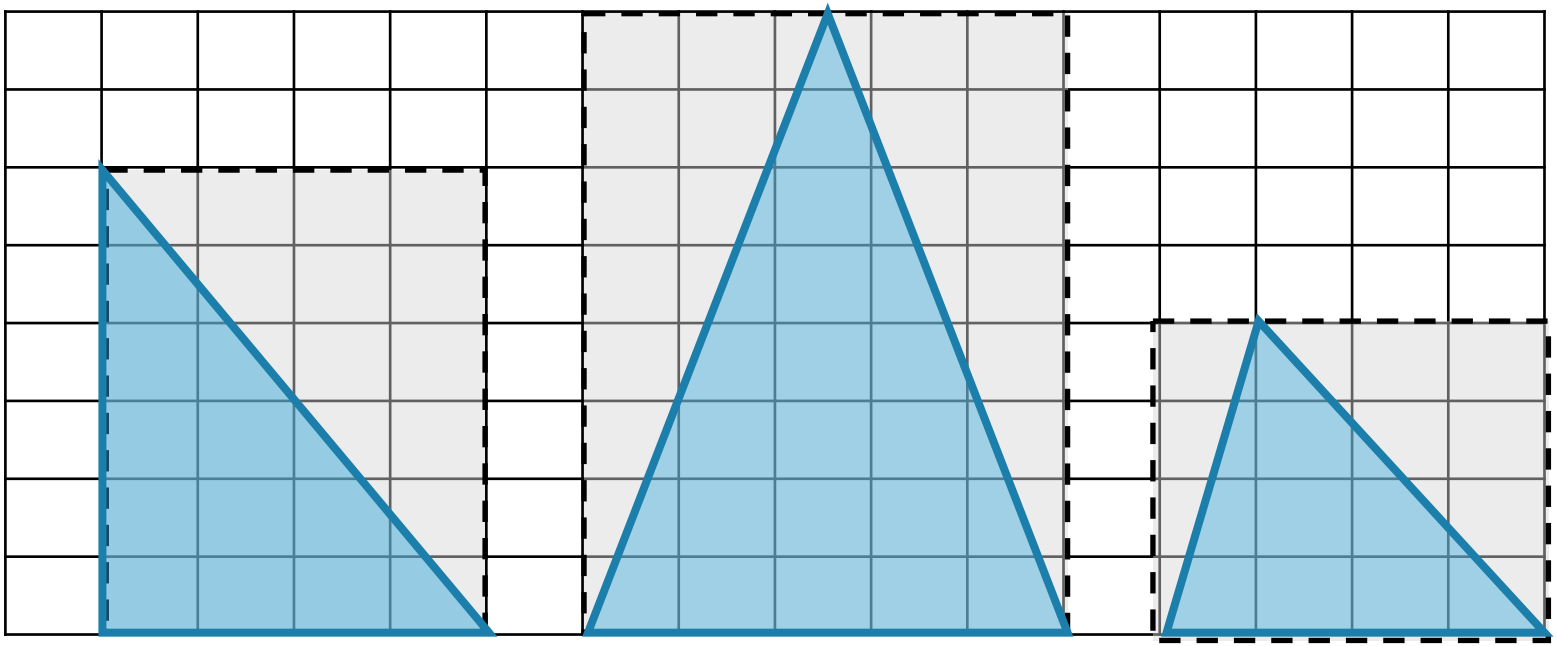
Dialogic Instruction

In the dialogic model, students must (a) actively engage in new mathematics, persevering to solve novel problems; (b) participate in a discourse of conjecture, explanation, and argumentation; (c) engage in generalization and abstraction, developing efficient problem-solving strategies and relating their ideas to conventional procedures; and to achieve fluency with these skills, (d) engage in some amount of practice.

Munter, Stein, and Smith 2015
(as cited in *Visible Learning 2017, p. 23*)

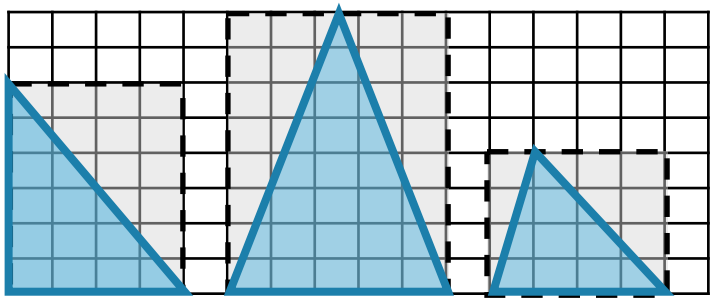
Turning the Lesson Sideways

- How can you determine the area of a triangle?
  - (prior knowledge – area of rectangles)



Poor Direct Instruction

How can you determine the area of a triangle?



I say the following to students and diagram what I'm saying on the board:

- *The area of a triangle is computed by the formula, ½ (base x height).*
- *The base can be any side of the triangle you choose, and the height is an altitude, or line from the vertex opposite the base drawn directly to the base and perpendicular to the base.*
- *Here are some examples of computing the area of triangles….*(I show the examples, doing the math myself and having students take notes.)
- *Now turn to page xx in your books and compute the areas of the triangles on that page.*

Good Direct Instruction

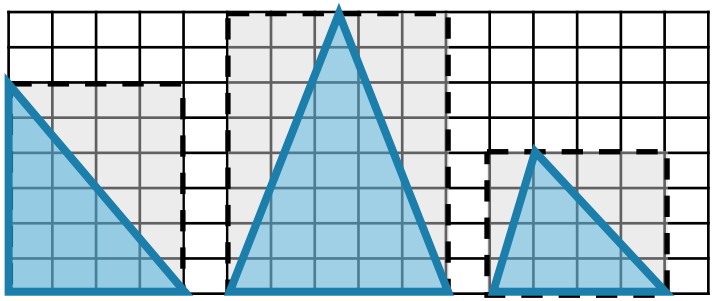
How can you determine the area of a triangle?

For a warm-up the class solves rectangle area problems. We process solutions. With the handout on the document camera, I say to students:

- *The goal today is for you to learn how to determine the area of triangles and connect this to what you know about finding the areas of rectangles..*
- I have students determine the area of the triangle by counting squares. Next is whole class share. I verify with students the area by counting squares.
- Next I describe the circumscribed rectangle around this triangle and have students determine the area of the rectangle using the formula they know. Then whole class share and clarification.
- Next, I have students determine the areas of the second triangle and related rectangle. Then whole class share and verification of areas.
- I ask students to, "*Look back out our two rectangle-triangle sets? What do you notice about the areas?*" Students talk to a partner for a minute to share ideas.
- Then we have a whole class share. I suggest, confirm, and verify as needed that the triangles are half the areas of the related rectangles.
- I show the class how to write this as a formula. We discuss connections of the triangle area formula to the rectangle area formula, and connect back to the diagram.
- I have students use the formulas to determine the areas in a third set of diagrams and to verify the triangle area by counting the squares to see that it is reasonable.
- Assign another problem to practice. Process whole class. Check for understanding.
- Assign class a set of problems to practice upon.

Poor Inquiry Lesson

How can you determine the area of a triangle?



I put a copy of the handout (the three triangles on grid) on the document camera to show the class. I say to the class,

- *You have a paper with some triangles and rectangles drawn on them.*
- *Determine the areas of the rectangles and triangles and write about what you discover or any patterns you notice.*

After students work for a while I have students present their answers, acknowledging strategies for determining the area of the triangles, and assign additional work.

Good Inquiry Lesson

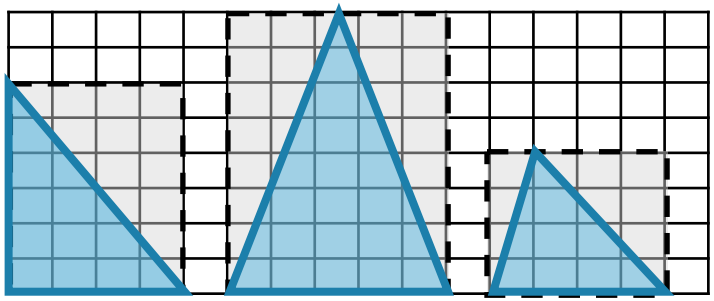
How can you determine the area of a triangle?

For a warm-up the class solves rectangle area problems. We process solutions.
With a copy of the handout on the document camera, I begin the lesson just as I did with the direct instruction model, going over the **first triangle – rectangle set**, with students doing much of the work as before, and I'm clarifying and correcting as needed.

- Then I start the inquiry portion of the lesson. There are four more sets of triangles with circumscribed rectangles on cards in each group. Each group member is to take one, copy it onto graph paper, and determine the areas of the triangle and rectangle.
- After about 4 minutes students Rotate cards, draw their new triangle and rectangle on the graph paper and determine the areas.
- After another 4 minutes I stop the class and give them 2 minutes to compare areas with the group members sitting next to them. Then check these whole class.
- Tell students to, *"Look for patterns in the areas of the triangles and rectangles, and write a conjecture about the areas of the triangles."* (2-3 minutes, assist as needed).
- Then students share and discus their groups. Teacher picks 2-3 conjectures to share with the class. Connects the conjectures to the work, and guides the conjectures into a formula for finding the area of the triangle.
- *Turn and talk to a partner for 30-60 seconds about how the area formula the triangle relates to the area formula for a rectangle and why this makes sense.*
- Some students share whole class. Teacher clarifies, confirms, and connects, making sure a clear connection is stated for all students to hear. The remainder of the lesson is similar to the direct instruction model.

Good Inquiry Lesson

How can you determine the area of a triangle?



Explicit Instruction	Direct Instruction	Inquiry-based
Instruction focuses on critical content.	✓	✓
Break down complex skills and strategies.	✓	✓
Provide systematic instruction.	✓	✓
Provide judicious practice.	✓	✓
Require frequent responses.	✓	✓
Monitor student performance closely.	✓	✓
Provide immediate affirmation and corrective feedback.	✓	✓
Deliver the lesson at a brisk pace.	✓	✓
Help students organize knowledge.	✓	✓

Teaching Challenges with Inquiry-based Instruction

- Providing mathematical tasks that are challenging at appropriate levels for students
- Supporting student work in a manner that guides students to reach the objective of the lesson while still grappling with the mathematics
- Making explicit the connections between facts, ideas, and procedures
- Checking for understanding and adjusting as needed

Teaching Challenges with Direct Instruction

- Balancing teacher talk with student engagement – Incorporating techniques to keep students active
- Providing questions that are challenging at appropriate levels for students
- Making explicit the connections between facts, ideas, and procedures
- Checking for understanding and adjusting as needed

When to Use Direct Instruction and When to Use Inquiry-based Instruction

- **Inquiry-based instruction**
  - Sensible connections are easily accessible
  - Patterns can lead to concepts
  - Setting the table (interest and understanding)
  - Continuing making math connections
- **Direct Instruction**
  - Laying or reviewing a foundation
  - Complex ideas
  - Often vocabulary and symbol meaning
  - Connecting ideas
  - Seeing examples

Direct Instruction or Inquiry-based Instruction?

When we talk about high-quality instruction, we're always asked the chicken-and-egg question: "Which comes first?" Should a mathematics lesson start with teacher-led instruction or with students attempting to solve problems on their own? Our answer: it depends. It depends on the learning intention. It depends on the expectations. It depends on students' background knowledge. It depends on students' cognitive, social, and emotional development and readiness.

Hattie, Fisher, and Frey 2017

Discourse

Mathematics discourse is defined as communication that centers on making meaning of mathematical concepts.

—Celedon-Pattichis & Ramirez, 2012

Mathematical Discourse

- Students explain their reasoning.
- Mistakes are opportunities for analysis.
- The final authority is the mathematics.

Include EL Students in Math Discourse

Research shows that ELs, even as they are learning English, can participate in discussions where they grapple with important mathematical content. Instruction for this population should not emphasize low-level language skills over opportunities to actively communicate about mathematical ideas.

—Moschkovich, 2012

Include EL Students in Math Discourse – Resources for Ideas

- ***Integrating the CA ELD Standards into K–12 Mathematics and Science Teaching and Learning***
  - WestEd/California Department of Education publication; **may need to google document title**
- ***Principles for the Design of Mathematics Curricula: Promoting Language and Content Development***
  - from Understanding Language/ Stanford Center for Assessment, Learning, and Equity
- The ***Universal Access*** section of the *California Mathematics Framework*

Discourse in Math

Math discussions differ from discourse/discussions in other content areas.

- In many content areas the point of discussion is often to express and support opinions and ideas.
- In math the point of discussion is usually to determine and prove a correct answer, or to recognize and describe mathematical relationships.

Mathematical Discourse and Higher-Order Questions

Higher-order questions generally challenge the student to provide additional information and engage in deeper understanding and reflection, and ultimately promote greater conceptual development.

—Nathan & Kim, 2007

Benefits of Everybody Writes

1. Improved thinking and understanding
2. Students remember twice as much
3. Every student participates
4. Select effective responses
5. Cold call on students
6. Guide students toward what is most important

—*Teach Like a Champion*, 2010

Steps for Meaningful Discourse

1. Initiate with a question or prompt that is focused on processes and/or outcomes that promote DOK 2–3.
2. Focus on the why behind the what.
3. Provide time to think.
4. Manage time to discuss.
5. Manage the process for sharing and connecting ideas.
6. Make mathematical connections explicit.
7. Always ask, *"Why does this make sense?"*

Promote and Manage Discourse Techniques

Questioning Strategies	Managing Techniques
• Collaborate on problem solving • Compare/contrast • Explain/justify/prove • Discuss disagreements • Investigate “what ifs” • Test student conjectures • Analyze errors • Predict • Connect ideas/concepts • ***Why does this make sense?***	• Clear directions • Wait time • Revoice • Restate • Revise • Add on • Think time • Pair-share • Think–write–pair-share • Snowball

Vocabulary

- You need words for discourse, and you use discourse to practice words. But which words?

- Math terms are generally what are called tier 3 words.

  Tier Three – Specialized words: Words that are specific within a certain field, such as math. For example, *linear, point, equation, variable*, etc.

  —Adapted from *Teaching Reading Sourcebook,* 2008

Why Is Vocabulary Important in Math?

- Research shows that reading comprehension positively affects achievement in arithmetic and problem solving.
- Vocabulary instruction should focus on specific words that are important to what students are learning (Marzano, 2001).
- Math vocabulary is confusing for many students for a variety of reasons that impede their ability to understand what they read and hear.

Understand Challenges with Math Vocabulary

- **Double Meanings:** Words mean different things in mathematical vs. nonmathematical contexts.
- **Multiple Terms:** More than one word can be used to describe the same concept.
- **Nested Vocabulary:** Some words require understanding of other math terms in order to be understood.
- **Symbol Intensity:** Math is full of symbols and graphic representations that carry as much weight as words.
- **Homophones:** Many math words sound like different nonmath words.
- **Small Words:** Many small words make a big difference in meaning.

Identify Challenging Words

Double Meanings:

table	fraction	even
base	rational	tangent
side	irrational	variable
point	operation	volume
mean	expression	etc.

Homophones:

cent	→	sent or scent
plane	→	plain
two	→	to or too
sum	→	some
sine	→	sign

Multiple Terms for Same Idea:

- altitude, height or length
- add, sum
- solve, answer, compute
- justify, explain, prove

Small Words or Phrases:

or	fewer	less than
many	then	increase
and	of	decrease
any	all	left

Unique Terms:

hypotenuse, parallelogram, coefficient, quadratic

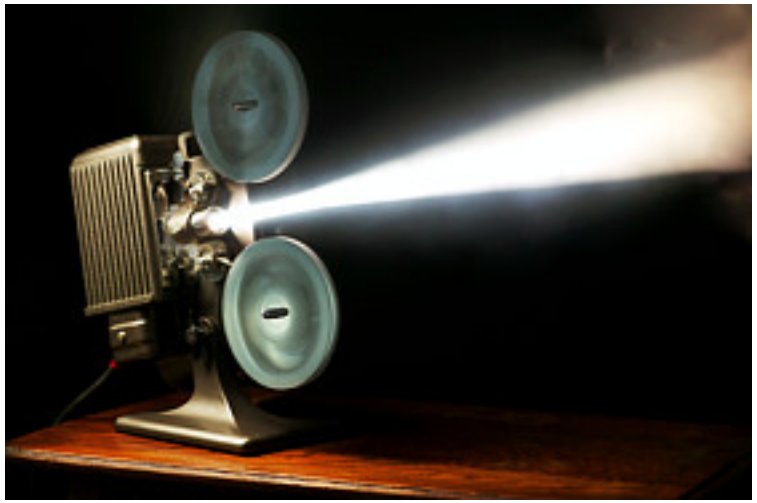
Similar Sounding Words:

tens vs. tenths
then vs. than
sixty vs. sixteen

Make Language Visible

Charts that help organize understanding and ideas about word:

- Math Word Wall
- Anchor Charts
- Frayer Model



Frayer Model

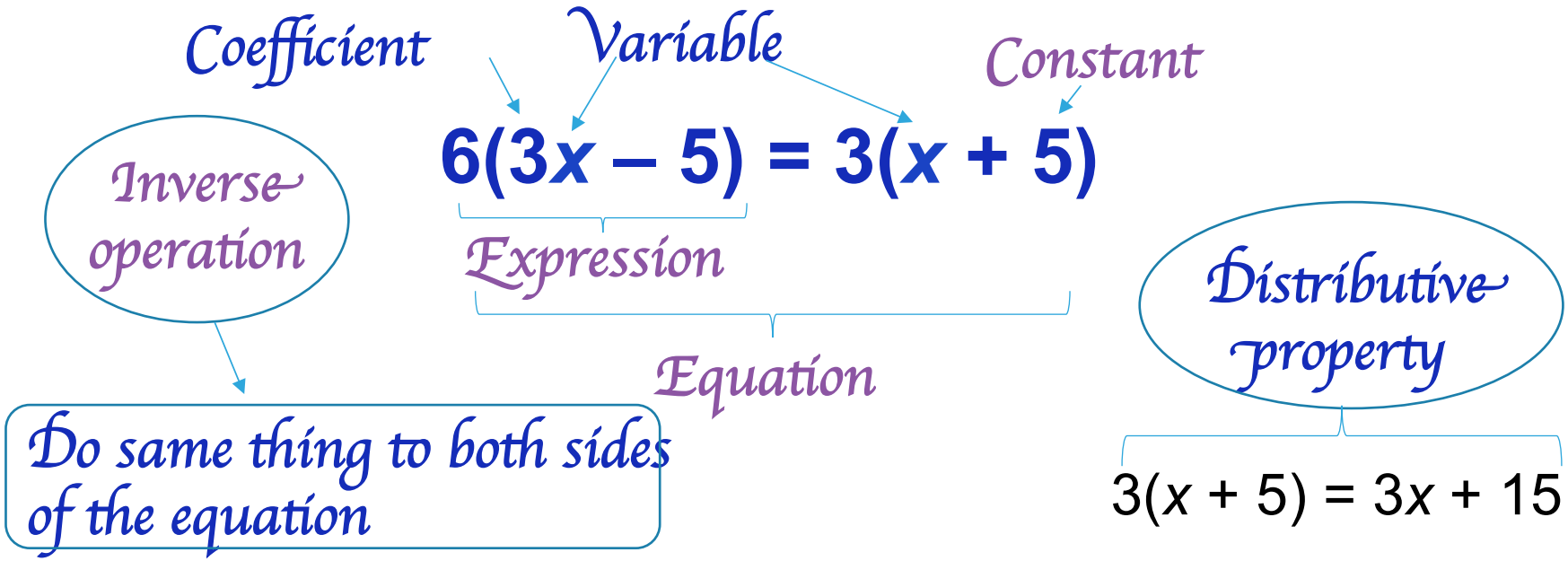
Definition (in own words)	**Facts/Characteristics**
Part of a whole. **One number divided by another number and written as one number over the other number.**	**Has a top number called the numerator, and a bottom number called the denominator.** **The denominator tells you how many parts the whole is split into.**
Examples	Nonexamples
$\frac{2}{5}, \frac{7}{4}$	5.12 50% 0

Fraction

Require Language to Be Used

What mathematical language or vocabulary is related directly to the following problem?

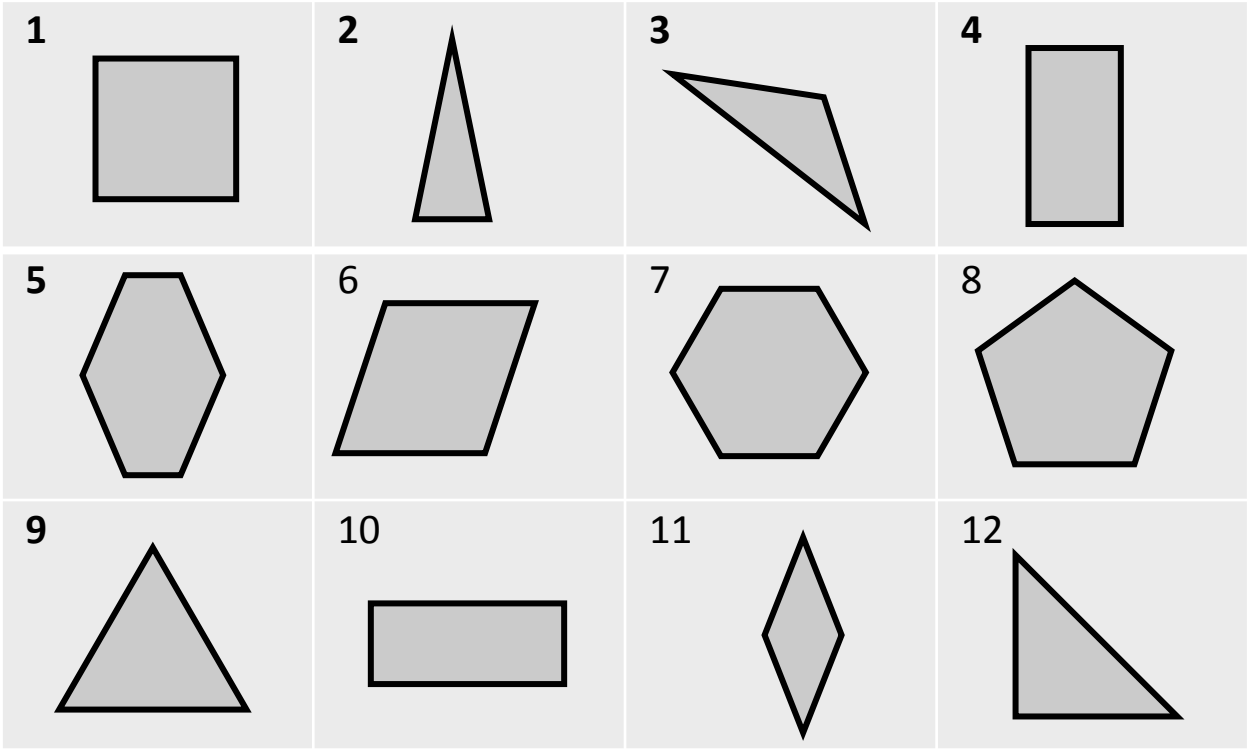
Solve:



Require Language Use- Guess My Polygon

Two players compete in the game using the polygon chart.

- Player 1 will pick a polygon from the chart but does not tell player 2 which polygon he/she chooses.
- Player 2 asks yes/no questions to figure out which polygon player 1 chose. The goal is to be able to know within 4–5 questions.



Vocabulary Wrap-Up

- Know the challenges with math vocabulary
- Teach math vocabulary in context
- Make math language visible
- Require that math language be used

Differentiation

Differentiation

*Consistent use of a variety of instructional approaches to modify **content**, **process**, and /or **products** in response to the learning readiness and interest of academically diverse students.*

(Tomlinson, 1995, p. 80)

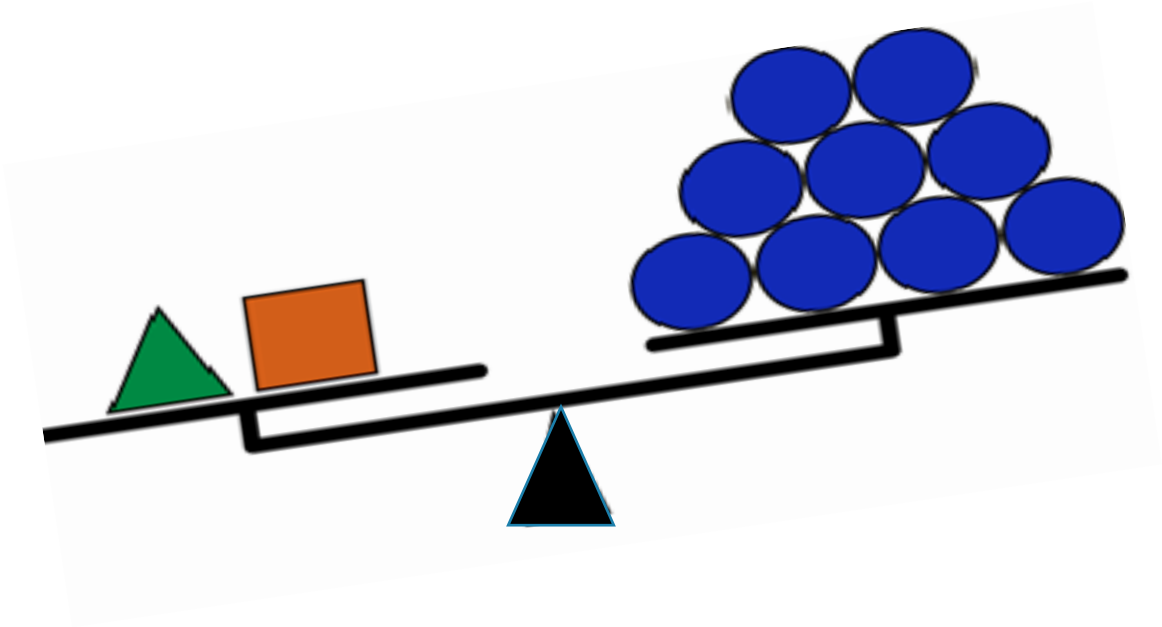
Content, Process, and Product

- **Adjusting content** is about scaffolding, depth, and/or applications.
- **Adjusting process** refers to adjusting the strategies students use to access information.
- **Adjusting product** means allowing some flexibility in which problems students solve and how understanding, solutions, or justifications are demonstrated.

Find a Balance Point

Differentiation is about finding a balancing point . . .

(Hattie et. al, 2017, p. 211)



Challenges with Differentiation

- Time
- Formative assessment
- Resources

Techniques Without Adding Time

- Questioning techniques
- Choice/menus
- Group/pair work
- Calling on students to share

Techniques that Add Time

- Peer tutoring (sometimes)
- Small group instruction
  - Flex days
  - Stations
- Other resources such as such as computer programs

Ideas We Explored Today

- Connecting explicit instructional techniques apply to both direct and inquiry-based instruction.
- Importance of and techniques for student discourse
- Why vocabulary is important in math and four key ideas to keep in mind.
- What are some differentiation techniques that work, with some that add little to no time to the lesson, and others that can take up the entire lesson.

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Let's Connect!

Dean Ballard

dballard@corelearn.com

corelearn.com

@COREInc

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