Effective Instruction in Mathematics: Engaging Direct Instruction and Guided Investigations

David J. Chard, Ph.D.

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Let's get started

- Working alone, create separate equations totalling 1-15 using only 4s and the basic operations in each equation.
- e.g., 11 = (4 x 4) – 4
- Discuss with a partner how you found your answers.

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Today's Agenda

- Define and Discuss Features Effective Instruction
- Review and Evaluate Lessons
- Discuss Questioning Approaches
- Review Evidence-based Practices for Tier 2 Instruction

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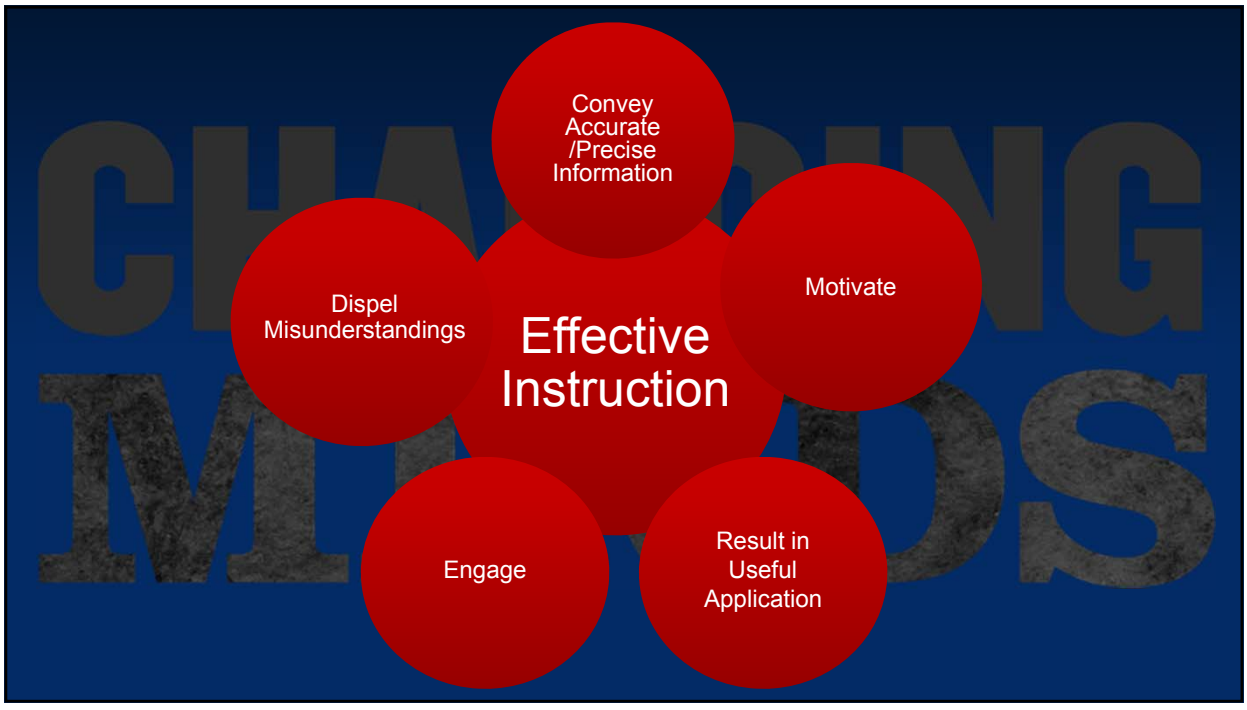
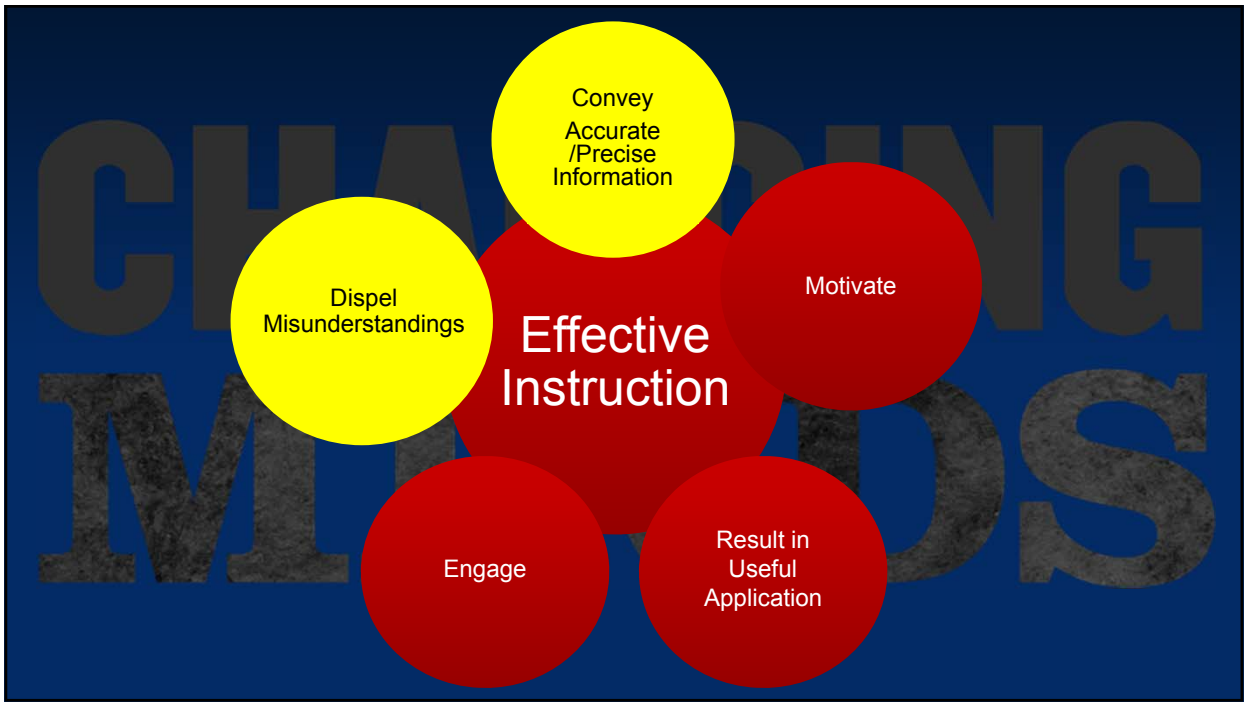
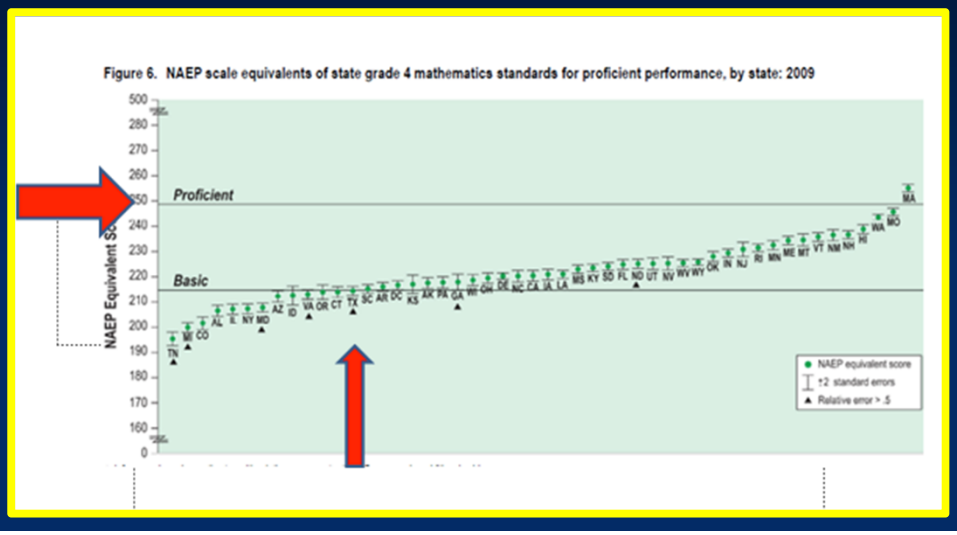




Raising Standards



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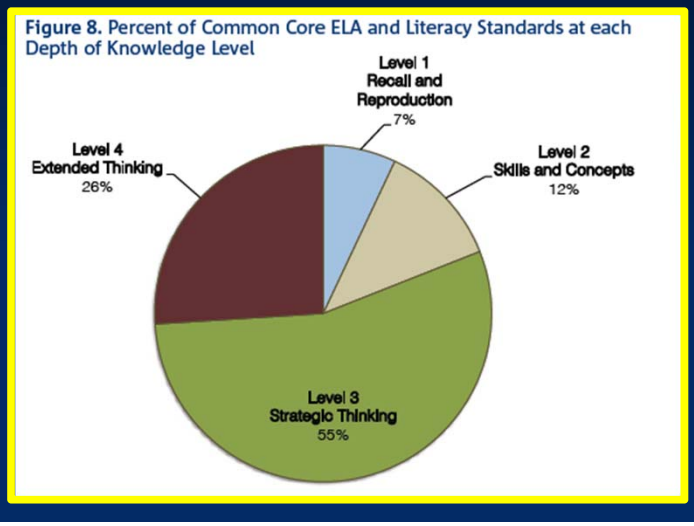
The Common Core Shift



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Instructional Shifts for the Common Core

Six Shifts in Math

- Focus
- Coherence
- Fluency
- Deep Understanding
- Applications
- Dual Intensity

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A closer look at the shifts

What do these mean to you?

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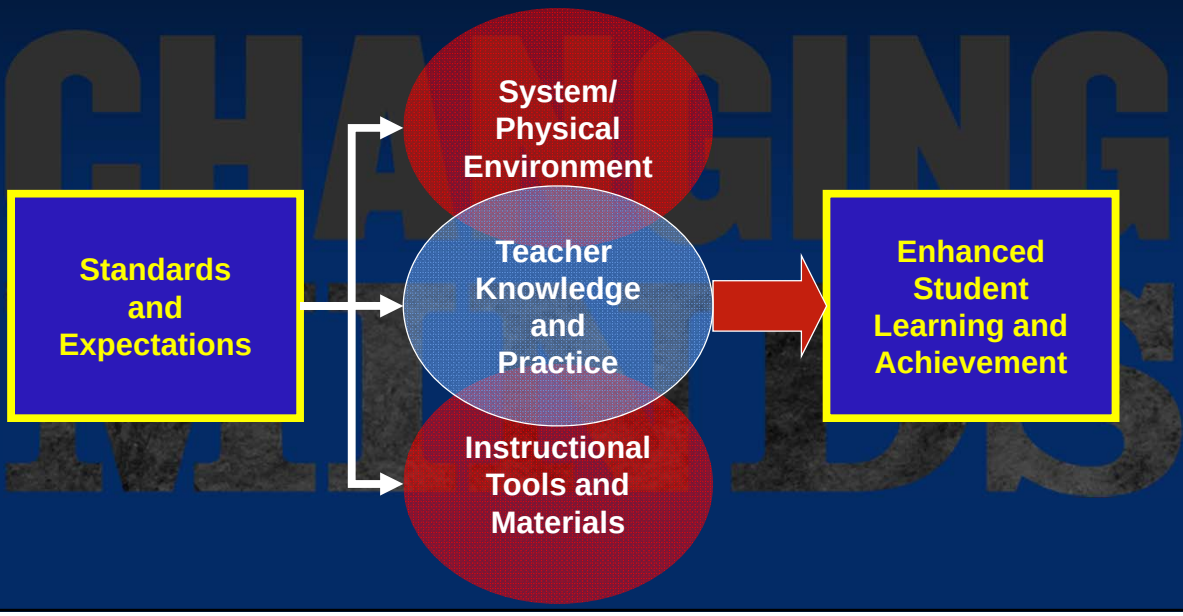
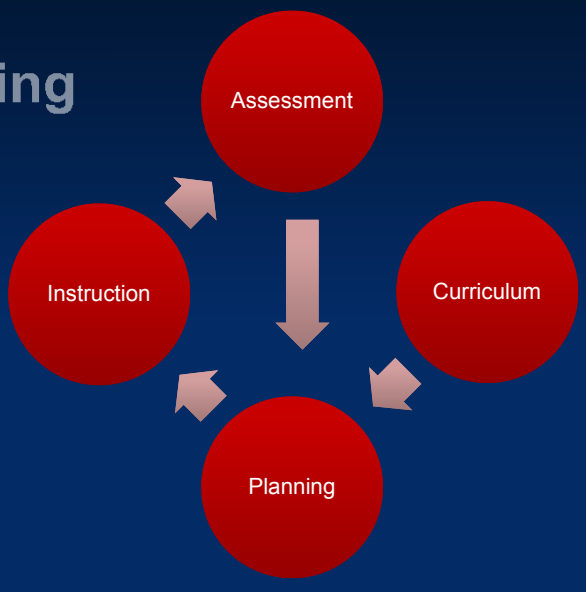
Shifts in Mathematics

Shift 1	Focus	Teachers use the power of the eraser and significantly narrow and deepen the scope of how time and energy is spent in the math classroom. They do so in order to focus deeply on only the concepts that are prioritized in the standards so that students reach strong foundational knowledge and deep conceptual understanding and are able to transfer mathematical skills and understanding across concepts and grades.
Shift 2	Coherence	Principals and teachers carefully connect the learning within and across grades so that, for example, fractions or multiplication spiral across grade levels and students can build new understanding onto foundations built in previous years. Teachers can begin to count on deep conceptual understanding of core content and build on it. Each standard is not a new event, but an extension of previous learning.
Shift 3	Fluency	Students are expected to have speed and accuracy with simple calculations; teachers structure class time and/or homework time for students to memorize, through repetition, core functions (found in the attached list of fluencies) such as multiplication tables so that they are more able to understand and manipulate more complex concepts.
Shift 4	Deep Understanding	Teachers teach more than "how to get the answer" and instead support students' ability to access concepts from a number of perspectives so that students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of core math concepts by applying them to new situations, as well as writing and speaking about their understanding.
Shift 5	Application	Students are expected to use math and choose the appropriate concept for application even when they are not prompted to do so. Teachers provide opportunities at all grade levels for students to apply math concepts in "real world" situations. Teachers in content areas outside of math, particularly science, ensure that students are using math – at all grade levels – to make meaning of and access content.
Shift 6	Dual Intensity	Students are practicing and understanding. There is more than a balance between these two things in the classroom – both are occurring with intensity. Teachers create opportunities for students to participate in "drills" and make use of those skills through extended application of math concepts. The amount of time and energy spent practicing and understanding learning environments is driven by the specific mathematical concept and therefore, varies throughout the given school year.

Powerful Core Mathematics Instruction



Cycle of Teaching



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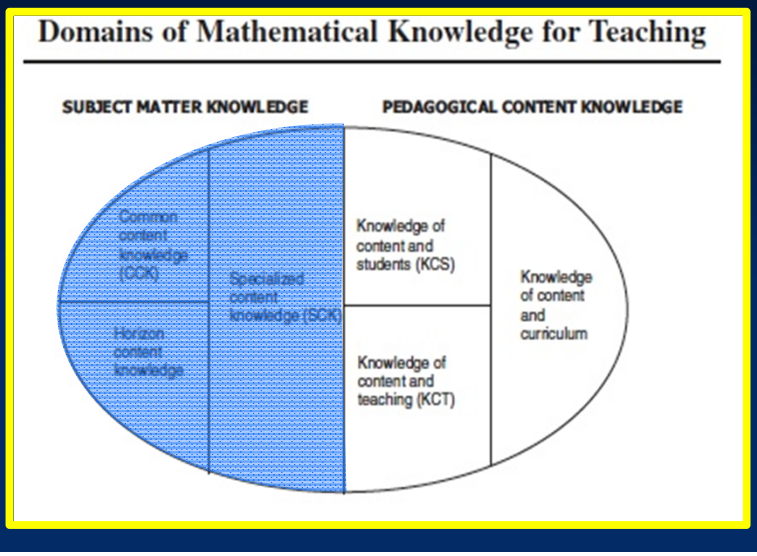
Types of Teacher Knowledge



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Let's Review A Lesson on Simple Interest

What to watch for:

- Approach to instructional design (e.g., what is introduced, how, when).
- Pace of instruction
- What assumptions are made about the learner.

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What Students Need to Know/Do	Tier 1 Design Considerations (UDL)
CCSS.Math.Content.6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems	• Identify core concept or principle
What I Need to Know	• Identify proximal pre-requisite knowledge students need to understand • List key vocabulary students will learn
What Do My Students Know?	• Develop scope of concrete to abstract representations to employ • Identify target problems

Concept or Principle	Simple Interest: The idea that if money is borrowed, the borrower pays a particular percentage of the principal to the lender. I = (Principal)x(rate)x(time).
Pre-requisite Knowledge And Skills/Procedures	**Concrete Representations**
	Semi-Concrete Representations
	Abstract Representations
Key Vocabulary	Target Problem Type You buy a pair of jeans for $110 that you've borrowed from a friend. You promised to pay your friend back with 10% interest calculated per month. How much will you owe if you pay it off after two months?

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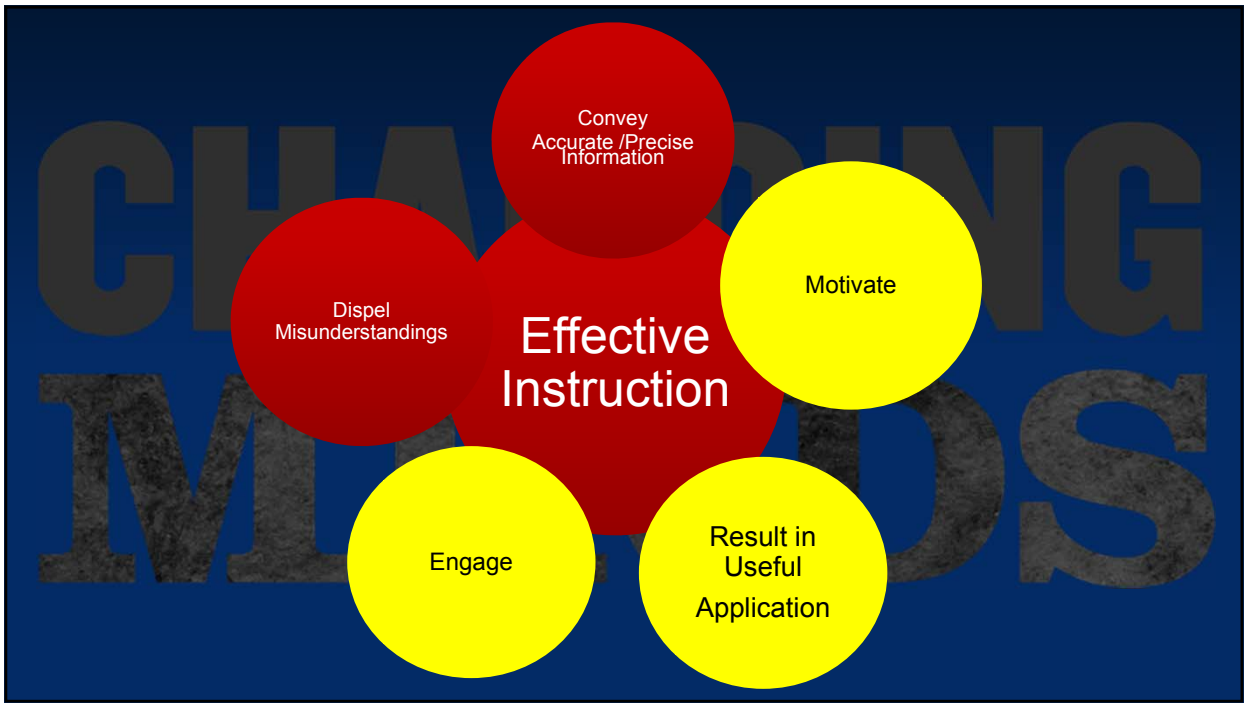

Process for Developing Differentiation

1. What you want students to know and be able to do, to reach the standards.
2. Review/collect assessment data (both class and student profiles) to identify:
   * readiness range
   * interests
   * learning preferences
3. Identify the task/activity in relation to desired learning outcome:
   * concept understanding
   * skill application
   * generalization of concept/skill
4. Determine the ***complexity*** of the task/activity, etc.

   Not very — somewhat — very
5. Design multiple versions of the tasks and/or activities; remain focused key concepts/skills; provide varied levels of challenge.
6. Match appropriate tasks to the students.
7. Monitor students' performance during the task/activity.
8. Conduct formative assessment of students' learning.
9. Adjust learning tasks/activities to accommodate students' needs.

Some Considerations for Differentiating Instruction

CONCRETE ⟷ ABSTRACT
(ideas, materials)

SIMPLE ⟷ COMPLEX
(resources, research, skills, goals)

SINGLE FACET ⟷ MULTIPLE FACETS
(directions, solutions)

MORE STRUCTURED ⟷ MORE OPEN-ENDED
(solutions, approaches, decisions)

LESS INDEPENDENCE ⟷ GREATER INDEPENDENCE
(planning, designing, monitoring)

Thinking about Differentiating a Lesson

- Identify the outcomes/learning expectations for this lesson/activity.
- Describe the lesson/activity you have designed for most students. Write it in the center column.
- How would you adjust the lesson/activity for students who need more support? Write it in the left column.
- How would you enhance the lesson/activity for students who need greater challenge? Write it in the right column.
- Use the differentiation considerations as you plan to adjust and enhance for students in your class.

readiness range of learners

ADJUST	MOST STUDENTS	ENHANCE

Motivation Strategies

- Address your students' negative beliefs about their mathematics abilities
- Get and keep your students' attention
- Build a mastery-oriented culture in your classroom

Motivation Strategies

Ego-Orientation	Mastery-Orientation
• Individual mathematics competition (e.g., math races) • Singular focus on getting it right • Focus on doing mathematics quickly • • •	• Collaborative problem-solving • Sharing solution strategies • Team-based competition focused on process and solutions • Providing feedback on process • • •

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Common Core Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

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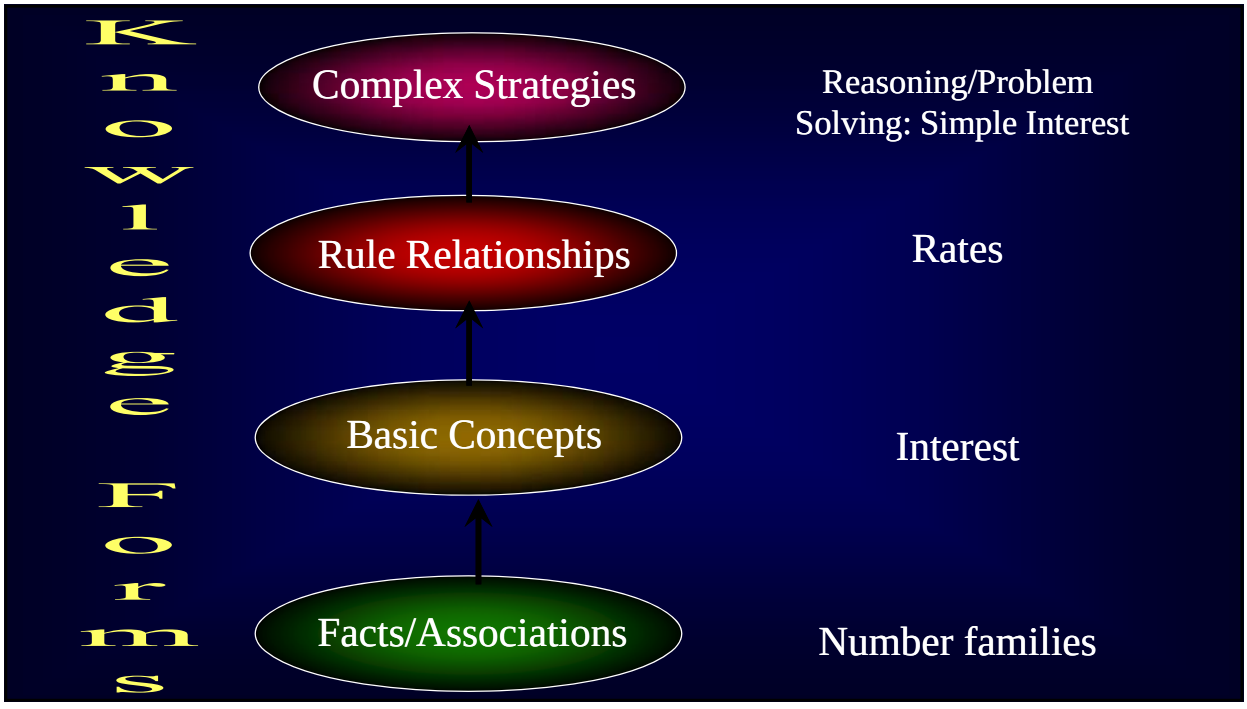

Question Approaches

IRE

Teacher: What is the principal in this problem?

Student: $1000

Teacher: Yes, the principal is $1000

Funneling

Teacher: This equation represents an interest problem. $I = $1000 x .20/yr. x 3 yrs. What is the interest rate?

Students: no response

Teacher: Remember rate is going to be a number per a unit time.

Student: .20/yr. is the rate.

Focusing

Teacher: This equation represents an interest problem. $I = $1000 x .20/yr. x 3 yrs. What is the interest rate?

Students: no response

Teacher: What do you think of when you think of interest rate?

Student: How much you have to pay the lender to borrow the money.

Teacher: What will happen to the interest if the lender wants to borrow it for 5 years?

Student: It will increase.

Questioning Examples

- What is the pattern of questions?
- What objective is achieved with this pattern of questions?
- When might this type of pattern be most useful?
- What problems might be associated with this type of questioning pattern?

Considerations about Questioning Patterns

- Different questioning patterns are useful for achieving different goals
- Students will benefit differentially to different questioning patterns based on what they have learned
- Focusing questions are most useful when students are working at the cognitive strategy level of learning

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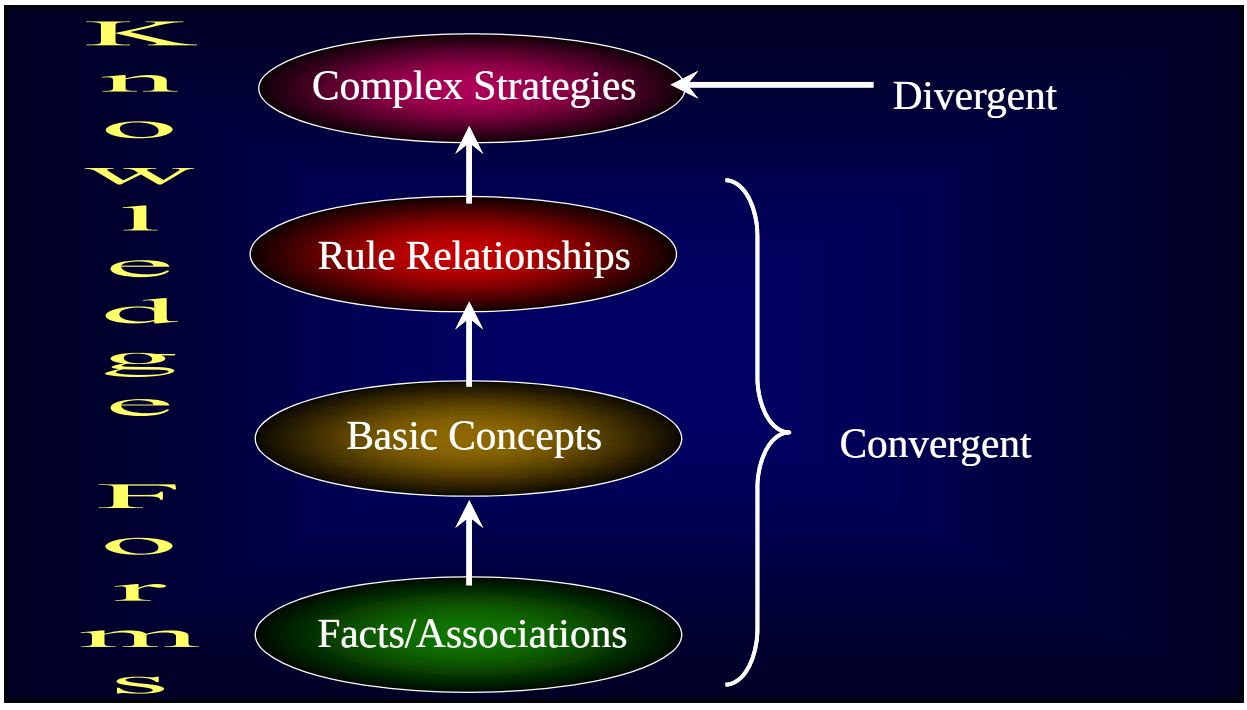
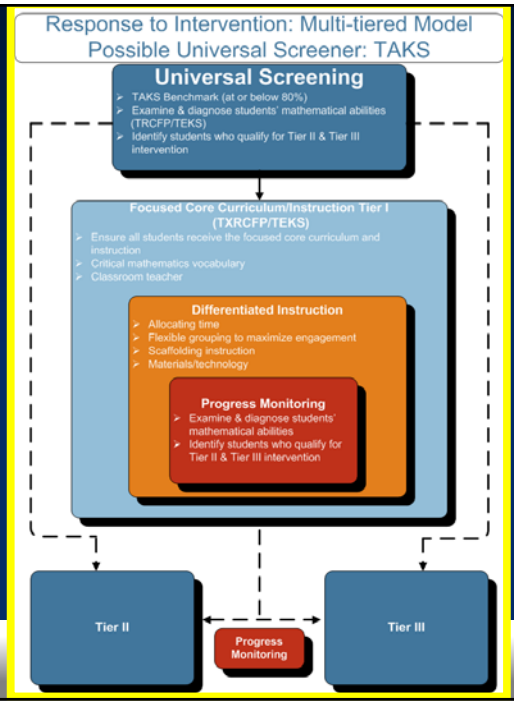


What Works for Struggling Learners Tier 2



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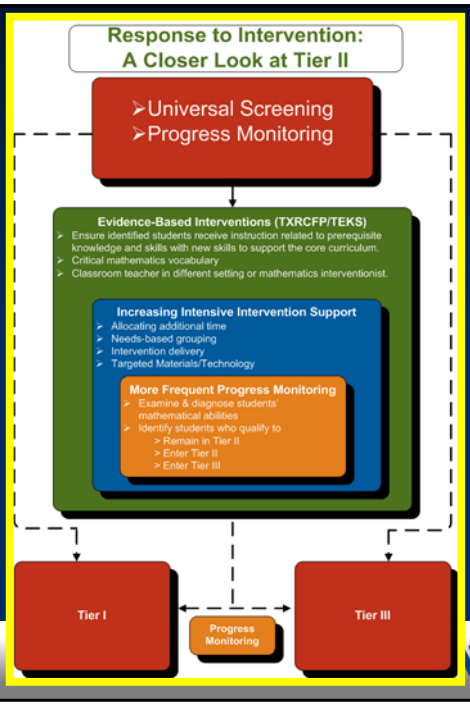
Focus on Tier 2



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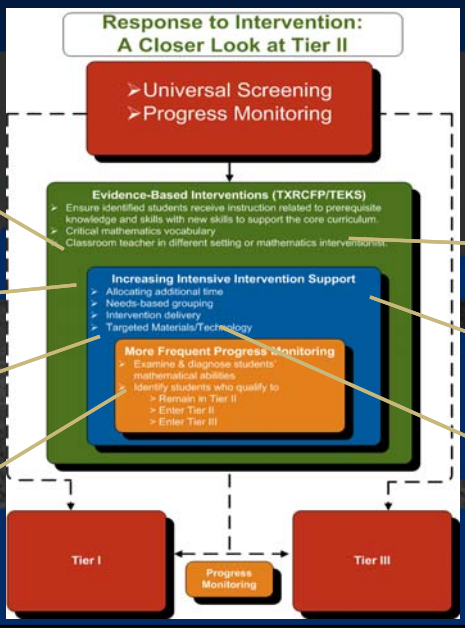
A Closer Look at Tier II



Focus

Interventionist

Grouping

Assessment

Program

Setting

Time

WHAT WORKS CLEARINGHOUSE

Recommendation	Level of evidence
Tier 1	
1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.	Moderate
Tiers 2 and 3	
2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.	Low
3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.	Strong
4. Interventions should include instruction on solving word problems that is based on common underlying structures.	Strong
5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.	Moderate
6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.	Moderate
7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.	Low

Explicit and Systematic Interventions

- Recommendation 3: Instruction during the intervention should be explicit and systematic including:
  - Clear models and demonstrations for solving problems,
  - Guided practice opportunities,
  - Student verbalizations,
  - Corrective feedback, and
  - Frequent cumulative review
- Level of evidence: Strong

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Learning Outcomes

- What did we learn?
  - Improved proficiency in solving word problems
  - Improved procedural fluency
- How might explicit and systematic instruction impact:
  - Students' connection between conceptual and procedural understanding?
  - Students' ability to generalize concepts and procedures?
  - Students' ability to communicate problem solving and reasoning?

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Teacher Models and Demonstrations

- Supplemental instruction should include explicit teacher modeling and demonstrations
- May include:
  - Teacher verbalization
  - Teacher demonstration of steps to solve problems

Guided Practice

- Supplemental instruction should gradually transition from teacher modeled problem solving to student directed problem solving
- May include:
  - Providing high to low levels of instructional support

Student Verbalization

- Supplemental instruction should include opportunities for students to talk aloud about the skills, knowledge, or problem solving procedures they are learning
- May include:
  - Strategies used to solve a problem
  - Explanation of solutions
  - Reasons supporting their decisions

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Corrective Feedback

- Supplemental instruction should be immediate and corrective
- Student should have opportunity to correct errors
- May include:
  - Description of what the student did correctly
  - Description of how the student can improve

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Cumulative Review

- Supplemental instruction should include sufficient practice that is both distributed and cumulative
- May include:
  - Practice using topics previously learned

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Lesson Observation

- Observe for explicit instruction
  - Modeling
  - Verbalization
  - Guided practice

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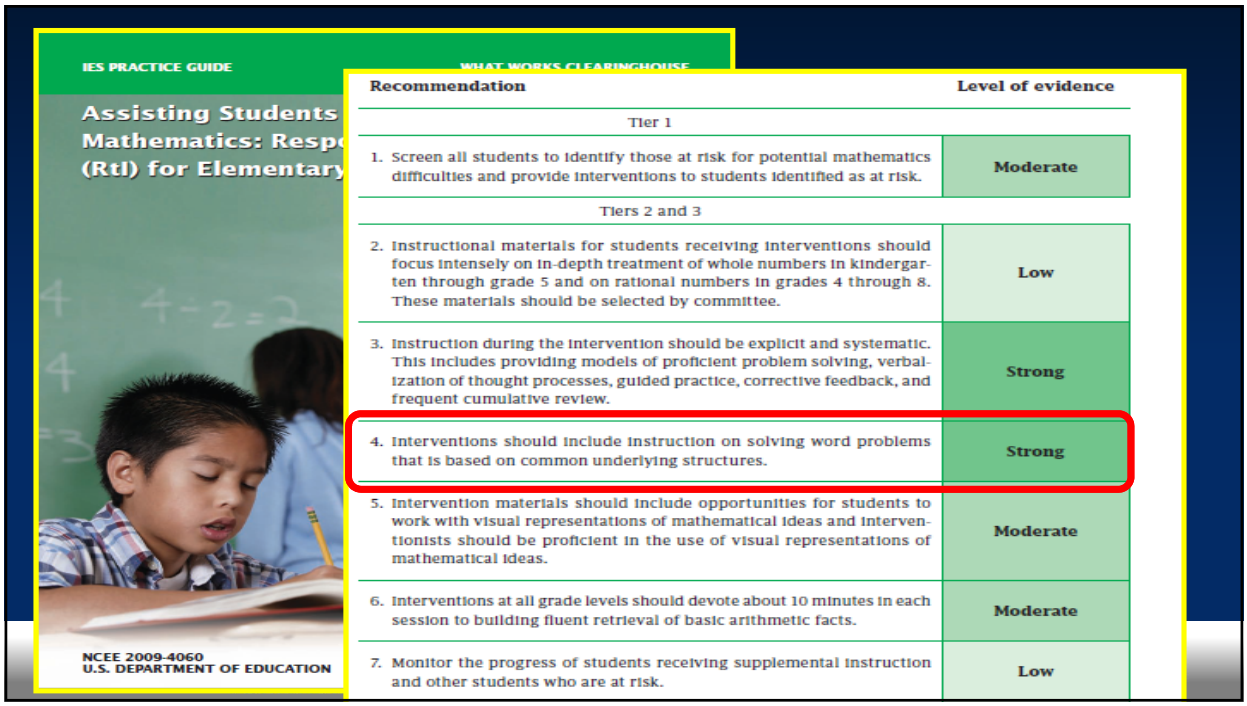

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6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.	Moderate
7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.	Low

Teaching Students about Problem Types

1. Explicitly teach problem types
   - Structure of different problem types
   - How to categorize problems based on type
   - How to determine appropriate solution methods for each problem type
2. Teach students to recognize problem types in unfamiliar problems and apply solution methods

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Resource for Teaching Problem Solving

- Schema-based Instruction
- Research conducted by Dr. Asha Jitendra and colleagues
- 4-step problem-solving
  1. Find the problem type
  2. Organize the information
  3. Plan to solve the problem
  4. Solve the problem

Jitendra, A. K. (2007). *Solving math word problems: Teaching students with learning disabilities using schema-based instruction*. Austin, TX: PRO-ED.

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7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.	Low
8. Include motivational strategies in tier 2 and tier 3 interventions.	Low

Using Visual Representations

- Recommendation 5: Intervention materials should include opportunities for students to work with visual representations of mathematical ideas. Interventionists should be proficient in the use of visual representations of mathematical ideas
- Level of evidence: Moderate

Using Visual Representations

- Intervention should include visual representations to model mathematical concepts
- May include:
  - Number lines
  - Graphs
  - Simple line drawings of concrete objects

Learning Outcomes

- What did we learn?
  - Improved achievement in general mathematics, pre-algebra concepts, word problems, operations
- How might visual representations impact:
  - Students' understanding of abstract symbols?
  - Students' strategies for solving word problems?
  - Students' conceptual understanding?

Considerations for Using Visual Representations

- Explicitly link the visual representation or model with the abstract mathematical symbol or concept
- Use consistent language across representations
- Provide ample practice opportunities
- Select examples and non-examples carefully

Instructional Sequence

- Supplemental instruction should present a systematic sequence of examples
- Examples should vary to include the range of skills and/or knowledge
- Possible sequence:
  - Introduce concept with concrete manipulatives
  - Reinforce concept with visual representations
  - Highlight relationship with mathematical symbols

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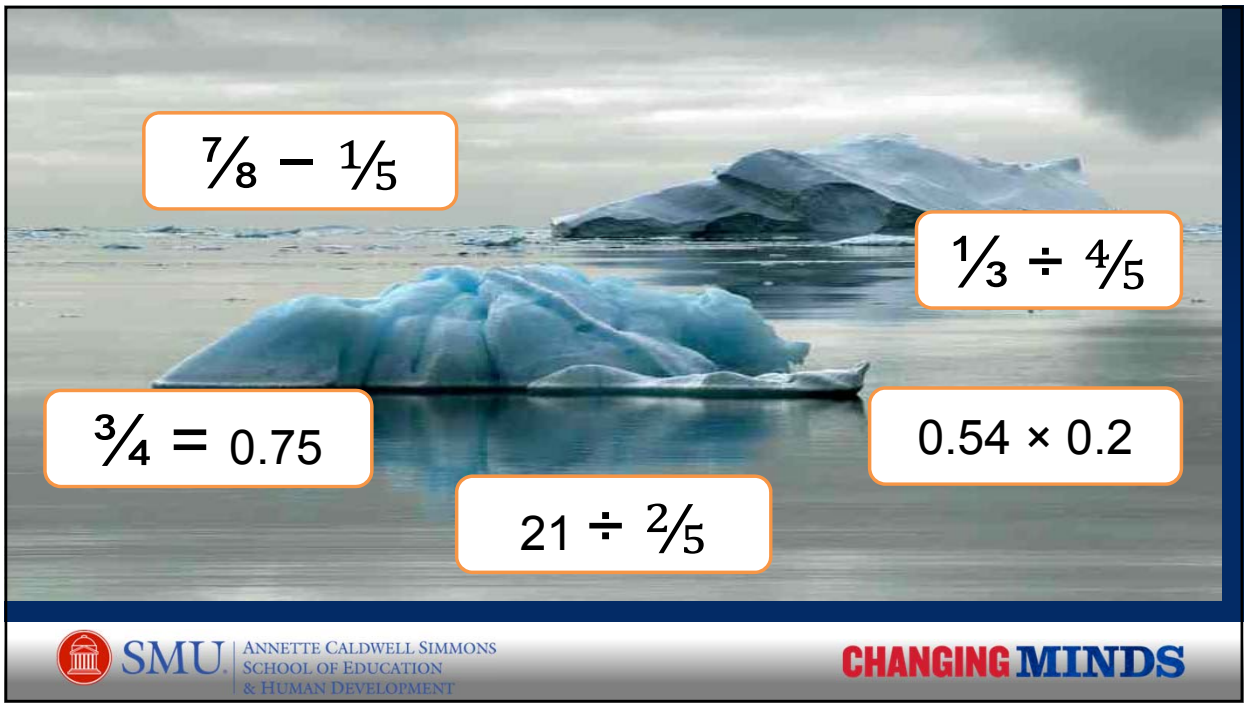

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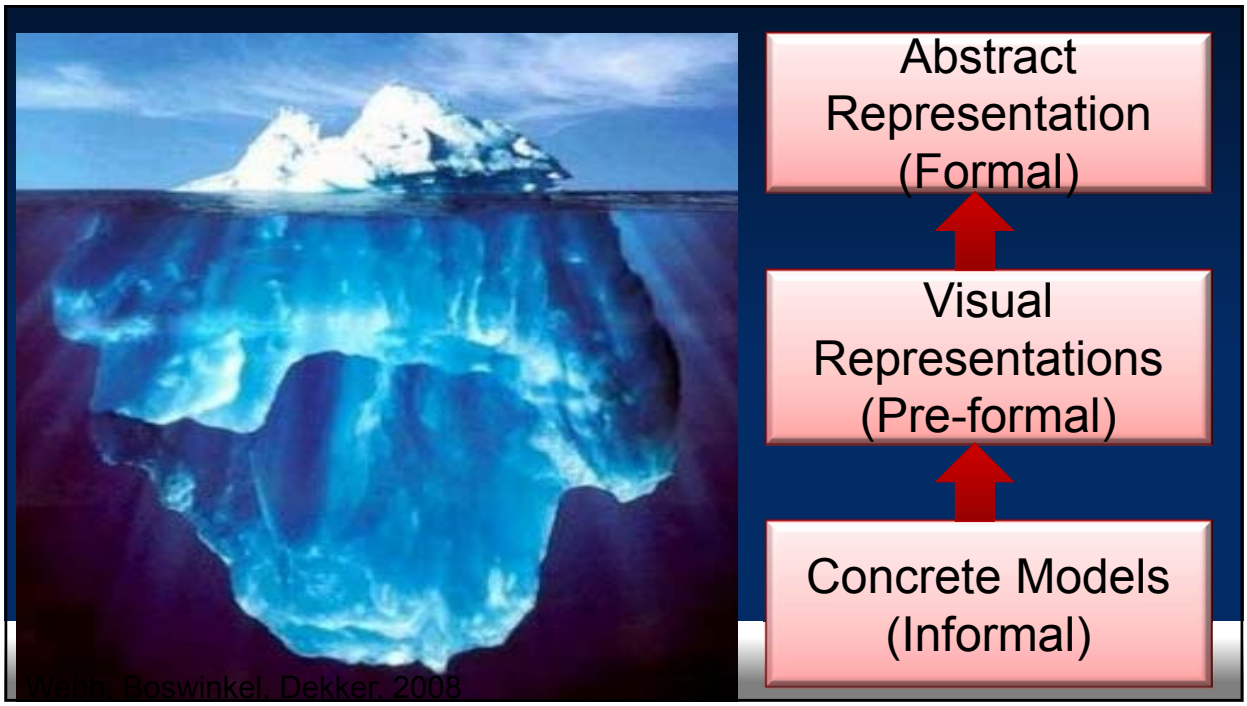
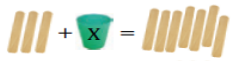
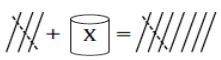


Example 8. A set of matched concrete, visual, and abstract representations to teach solving single-variable equations

3 + X = 7

Solving the Equation with Concrete Manipulatives (Cups and Sticks)	Solving the Equation with Visual Representations of Cups and Sticks	Solving the Equation with Abstract Symbols
A 		3 + 1X = 7
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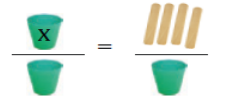
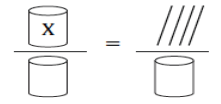
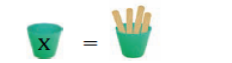
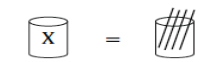
C 		1X / 1 = 4 / 1
D		
E 		X = 4

Concrete Steps
A. 3 sticks plus one group of X equals 7 sticks
B. Subtract 3 sticks from each side of the equation
C. The equation now reads as one group of X equals 4 sticks
D. Divide each side of the equation by one group
E. One group of X is equal to four sticks (i.e., 1X/group = 4 sticks/group; 1X = 4 sticks)

How might we support students' understanding of 'simple interest' visually?

Concrete?

Visual?

Abstract?

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3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.	Strong
4. Interventions should include instruction on solving word problems that is based on common underlying structures.	Strong
5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.	Moderate
6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.	Moderate
7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.	Low
8. Include motivational strategies in tier 2 and tier 3 interventions.	Low

Source: Authors' compilation based on analysis described in text.

Build Fluency of Arithmetic Facts

- Recommendation 6: Intervention at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.
- Level of evidence: Moderate

Build Fluency of Arithmetic Facts

- Intervention should provide about 10 minutes per session of instruction to build quick retrieval of basic arithmetic facts and number sense.
- May include:
  - Technology based activities
  - Number talks
  - Other materials for extensive, strategic practice to facilitate arithmetic mastery

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Learning Outcomes

- What did we learn?
  - Students ability to quickly retrieve arithmetic facts is associated with later mathematics success.
- How might fluency with numbers and operations impact:
  - Students' understanding of abstract symbols?
  - Students' strategies for solving word problems?
  - Students' conceptual understanding?

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References

Boaler. J. (2008). *What's math got to do with it?* New York: Viking.

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research, 79*, 1202-1242.

Herbel-Eisenmann, B.A., & Breyfogle, M.A. (2005). Questioning our patterns of questioning. *Mathematics Teaching in the Middle School, 10*, 484-489.

Hill, H.C., Rowan, B., & Ball, D.L. (2005). Effects of teacher mathematics knowledge for teaching on student achievement. *American Educational Research Journal, 42*, 371-406.

Ketterlin Geller, L. R., Baker, S., & Chard, D. J. (2008). Mathematics instruction and assessment in secondary settings. In A. Thomas & J. Grimes (Eds.). *Best Practices in School Psychology* (pp. 453-464). Bethesda, MD: National Association of School Psychologists.

Ketterlin Geller, K., & Chard, D. J. (2011). Algebra readiness for students with learning difficulties in grades 4-8: Support through the study of number. *Australian Journal of Learning Difficulties, 16* (1), 65–78.

Thomlinson, C.A (2001). *How to differentiate instruction in mixed-ability classrooms (2nd Edition)*. Alexandria, VA: ASCD.

SMU Annette Caldwell Simmons School of Education & Human Development

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