Problem Solving in Elementary Math

Participant Handout
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CORE Mission

CORE serves as a trusted advisor at all levels of preK–12 education, working collaboratively with educators to support literacy and math achievement growth for all students.

Our implementation support services and products help our customers build their own capacity for effective instruction by laying a foundation of research-based knowledge, supporting the use of proven tools, and developing leadership.

As an organization committed to integrity, excellence, and service, we believe that with informed school and district administrators, expert teaching, and well-implemented programs, all students can become proficient academically.
IES Practice Guide: 
Improving Mathematical Problem Solving in Grades 4 Through 8

1. **Prepare problems and use them in whole-class instruction.**
   - Include both routine and nonroutine problems in problem-solving activities.
   - Ensure that students will understand the problem by addressing issues students might encounter with the problem’s context or language.
   - Consider students’ knowledge of mathematical content when planning lessons.

2. **Assist students in monitoring and reflecting on the problem-solving process.**
   - Provide students with a list of prompts to help them monitor and reflect during the problem-solving process.
   - Model how to monitor and reflect on the problem-solving process.
   - Use student thinking about a problem to develop students’ ability to monitor and reflect.

3. **Teach students how to use visual representations.**
   - Select visual representations that are appropriate for students and the problems they are solving.
   - Use think-alouds and discussions to teach students how to represent problems visually.
   - Show students how to convert the visually represented information into mathematical notation.

4. **Expose students to multiple problem-solving strategies.**
   - Provide instruction in multiple strategies.
   - Provide opportunities for students to compare multiple strategies in worked examples.
   - Ask students to generate and share multiple strategies for solving a problem.

5. **Help students recognize and articulate mathematical concepts and notations.**
   - Describe relevant mathematical concepts and notation, and relate them to the problem-solving activity.
   - Ask students to explain each step used to solve a problem in a worked example.
   - Help students make sense of algebraic notation.

Table 1. Common Addition and Subtraction Situations

<table>
<thead>
<tr>
<th></th>
<th>Result Unknown</th>
<th>Change Unknown</th>
<th>Start Unknown</th>
</tr>
</thead>
</table>
| **Add To**           | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?  
2 + 3 = ? | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?  
2 + ? = 5 | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?  
? + 3 = 5 |
| **Take From**        | Five apples were on the table. I ate two apples. How many apples are on the table now?  
5 – 2 = ? | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?  
5 – ? = 3 | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?  
? – 2 = 3 |

<table>
<thead>
<tr>
<th><strong>Total Unknown</strong></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| **Put Together/ Take Apart** | Three red apples and two green apples are on the table. How many apples are on the table?  
3 + 2 = ? | Five apples are on the table. Three are red and the rest are green. How many apples are green?  
3 + ? = 5, 5 – 3 = ? | Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?  
5 = 0 + 5, 5 = 5 + 0  
5 = 1 + 4, 5 = 4 + 1  
5 = 2 + 3, 5 = 3 + 2 |

<table>
<thead>
<tr>
<th><strong>Difference Unknown</strong></th>
<th><strong>Bigger Unknown</strong></th>
<th><strong>Smaller Unknown</strong></th>
</tr>
</thead>
</table>
| (“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?  
2 + ? = 5, 5 – 2 = ? | (Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?  
(Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have?  
2 + 3 = ?, 3 + 2 = ? | (Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?  
(Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have?  
5 – 3 = ?, ? + 3 = 5 |

Table 2. Common Multiplication and Division Situations

<table>
<thead>
<tr>
<th></th>
<th>Unknown Product</th>
<th>Group Size Unknown (&quot;How many in each group?&quot; Division)</th>
<th>Number of Groups Unknown (&quot;How many groups?&quot; Division)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Equal Groups</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Arrays, Area</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td><strong>Compare</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>General</strong></td>
<td></td>
<td></td>
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</tr>
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<tr>
<td>$3 \times 6 = ?$</td>
<td>$3 \times ? = 18, \ &amp; \ 18 \div 3 = ?$</td>
<td>$? \times 6 = 18 \ &amp; \ 18 \div 6 = ?$</td>
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<td><strong>Equal Groups</strong></td>
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<td><strong>General</strong></td>
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</table>

Solving Problems B1 (K–2)

Grades K–2: Solve each problem and identify the problem type. Then rewrite the problem to create a different problem type.

1. Sheila took all the dollar bills out of her two pockets. The total was $9. How much did she have in each pocket? Show how you know the answer is correct.

2. Ricky had 6 jelly beans and his brother gave him 5 more jelly beans. How many jelly beans does Ricky have now? Show how you got the answer.

3. Denise had 24 cookies on a tray. Her friends came over and ate some of the cookies. Now there are only 9 cookies left. How many cookies did Denise’s friends eat? Use a diagram to show how to solve the problem.
Solving Problems B1 (3–5)

Grades 3–5: Solve each problem and identify the problem type. Then rewrite the problem to create a different problem type.

1. Larry bought 8 boxes of food bars. Each box has 12 food bars. How many food bars did Larry buy altogether? Use a diagram to prove your answer is correct.

2. Susan needs 108 small bottles of orange juice for the field trip. The orange juice is sold with 4 bottles in each package. How many packages of orange juice does Susan need to buy? Show how you determined the answer.

3. Mickey has three times as much money as Jimmy. Altogether Mickey and Jimmy have $36. How much money does Jimmy have? Explain/show how you got the answer.
Examples of Problem Solving As a Process for Learning

1. Review one or two of the sample lessons described on the following two pages.

2. What is the mathematics students will learn through each lesson?

______________________________________________________________________________

______________________________________________________________________________

______________________________________________________________________________

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______________________________________________________________________________
### Understand How Numbers Relate to 5 and 10

Students visualize numbers by seeing them concretely on five and ten frames and determine how much less or how much more numbers are than 5 or 10.

<table>
<thead>
<tr>
<th>Grades K–1</th>
<th>CCSSM K.CC.5, K.OA.4,5; 1.NBT.2a,b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students visualize numbers by seeing them concretely on five and ten frames and determine how much less or how much more numbers are than 5 or 10.</td>
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</tbody>
</table>

- Give each student a five or ten frame and some chips. Call out a number and have students place that number of chips on their five or ten frame. Ask students to describe what their number looks like. Some may put chips in order, and some may have spaces in between. It is only important that they correctly show the quantity.

- Next, ask students to tell you how many more chips would be needed to fill the five or ten frame—to make a 5 or 10. In other words, how far is their number from 5 or 10?

- Additionally with a five or ten frame, students can show numbers greater than 5 or 10. For example, on a five frame, a student can show 8 as a five and three more chips on the side. With a ten frame, a student can show 12 as a full ten and two more chips on the side. This builds understanding of numbers in relationship to these benchmark values.

### Commutative Property of Addition

Students realize the commutative property of addition and that it does not apply to subtraction.

- Use concrete objects in each case to explore and analyze.

- Students compare pairs of numbers added together, such as $3 + 5$ and $5 + 3$, $2 + 4$ and $4 + 2$, etc., and conjecture about whether or not the sum is always the same, even if they switch the order of the addends.

- Students compare switching the numbers in a subtraction problem, such as $5 - 3$ and $3 - 5$, to see if the difference changes or remains the same.

- In a whole-class discussion, clarify and make clear the mathematics property.
<table>
<thead>
<tr>
<th>Grade 4</th>
<th>CCSSM 4.NF.1</th>
<th>Grade 5</th>
<th>CCSSM 5.MD.5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Creating Equivalent Fractions</strong></td>
<td><strong>Volume of Rectangular Prisms (Boxes)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students determine how equivalent fractions are related numerically and link this to prior knowledge of multiplying any number by 1.</td>
<td>Students determine a shortcut or formula for the volume of a rectangular box, as well as a general formula they can use later with all prisms and cylinders.</td>
<td></td>
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</tr>
<tr>
<td>• Students compare selected fractions that they know are equal based on use of a tool such as fraction strips.</td>
<td>• Students work in pairs or small groups filling boxes with cubes.</td>
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</tr>
<tr>
<td>• Students write equations showing the equal fractions, such as $\frac{1}{3} = \frac{2}{6}$, $\frac{1}{3} = \frac{3}{9}$.</td>
<td>• They begin with smaller boxes that can be entirely filled with the cubes they have available.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• After looking at several such pairs, students identify a pattern in the relationships between the numbers in equal fractions and generalize this into a rule: “If you multiply the numerator and denominator by the same number, you get an equivalent fraction.”</td>
<td>• Next, they try a box they can only partially fill, covering the base and maybe a couple layers above the base. They have to predict how many layers of the base it would take to fill the box to the top.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• (“If you multiply the numerator and denominator both by 2, that is the same as multiplying the fraction by $\frac{2}{2}$, and we know $2/2 = 1$. What do we know about multiplying something by 1?”)</td>
<td>• Whole-class processing of this work should lead to two important formulas: 1) general formula for volume of prisms: area of base $\times$ height; 2) specific formula for volume of rectangular prisms, $L \times W \times H$.</td>
<td></td>
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</table>

**Directions for teacher:** Provide jars with 15–20 objects of different sizes and shapes. Repeat this activity many times over the course of the year, varying the number and types of objects in the jar. As students become proficient with 15–20 objects, increase the number of objects to 30–35.

**Directions for students:**

1. Count the number of objects in the jar.
2. Make a drawing that represents this number of objects.
3. Create an equivalent number of objects (use any other objects you want).

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**CORE Grade 1**

1. Henry has 11 pennies and Marlene has 24 pennies. How many more pennies does Marlene have than Henry?

2. Suppose Henry actually has 11 dimes and Marlene has 24 pennies. How can they split the money up equally so they both have the same amount? Show how you determined the answer. Use drawings and/or words.
The perimeter of the rectangular state park shown is 42 miles.

A ranger estimates that there are 9 deer in each square mile of the park.

If this estimate is correct, how many total deer are in the park? Explain your answer using numbers, symbols, and words.

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Mr. Torres sold a total of 30 boxes of sports cards at his store on Monday. These boxes contained only baseball cards and football cards.

- Each box contained 25 sports cards.
- He earned $3 for each sports card he sold.
- He earned a total of $1,134 from the football cards he sold.

What amount of money did Mr. Torres earn from the baseball cards?
Problem Solving in Elementary Math

Objectives

- Recognize problem solving as a tool for learning and applying math and as a goal of learning in itself.
- Analyze problem-solving activities including sample assessments from the Partnership for Assessment of Readiness for College and Careers (PARCC) and Smarter Balanced Assessment Consortium (SBAC).

Activity: Ordering Cards Problem

- Closely watch the card trick performed by the facilitator.
- Work individually or in pairs to figure out how to correctly arrange 10 cards, numbered 1–10, so that you can perform the card trick.
- What strategies did you use to figure it out?
- What does this have to do with problem solving?

Define Problem Solving

Problem solving involves reasoning and analysis, argument construction, and the development of innovative strategies. These abilities are used not only in advanced mathematics topics—such as algebra, geometry, and calculus—but also throughout the entire mathematics curriculum beginning in kindergarten.

Institute of Education Sciences (IES), Improving Mathematical Problem Solving in Grades 4–8, 2012
Recommendations: IES Guide on Improving Problem Solving

1. Prepare problems and use them in whole-class instruction.
2. Assist students in monitoring and reflecting on the problem-solving process.
3. Teach students how to use visual representations.
4. Expose students to multiple problem-solving strategies.
5. Help students recognize and articulate mathematical concepts and notations.

Institute of Education Sciences (IES), Improving Mathematical Problem Solving in Grades 4–8, 2012

Three Perspectives on Problem Solving

- **Problem solving as a goal**: Learn about how to problem solve.
- **Problem solving as a process**: Extend and learn math concepts through solving selected problems.
- **Problem solving as a tool for applications and modeling**: Apply math to real-world or word problems, and use mathematics to model the situations in these problems.

IES Sample of Steps for Problem Solving

1. Identify the givens and goals of the problem.
2. Identify the problem type.
3. Recall similar problems to help solve the current problem.
4. Use a visual to represent and solve the problem.
5. Solve the problem.
6. Check the solution.

Institute of Education Sciences (IES), Improving Mathematical Problem Solving in Grades 4–8, 2012
Research on Problem Solving

Schoenfeld in his 1992 review of the literature concluded that attempts to teach students to use general problem-solving strategies (e.g., draw a picture, identify the givens and goals, consider a similar problem) generally had not been successful. He recommended that better results might be obtained by developing and teaching more specific problem-solving strategies (that link more clearly to classes of problems) . . .

Lesh & Zawojewski 2007

Multiplication & Division Problem Types

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<td><strong>General</strong></td>
<td>a × b = ? a × ? = p, p ÷ a = ?</td>
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Addition & Subtraction Problem Types

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<td>2 + ? = 5</td>
</tr>
<tr>
<td>Take From</td>
<td>5 – 2 = ?</td>
<td>5 – ? = 3</td>
</tr>
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<td><strong>Total Unknown</strong></td>
<td><strong>Addend Unknown</strong></td>
<td><strong>Both Addends Unknown</strong></td>
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<tr>
<td>Put Together/Take Apart</td>
<td>3 + 2 = ?</td>
<td>3 + ? = 5, 5 – 3 = ?</td>
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CCSSM 2010

Solving Problems – B1

- **K–2**: Sheila took all the dollar bills out of her two pockets. The total was $9. How much did she have in each pocket? Show how you know the answer is correct.

- **3–5**: Mickey has three times as much money as Jimmy. Altogether Mickey and Jimmy have $36. How much money does Jimmy have? Explain/show how you got the answer.
Problem Solving As a Process/Method

Problem solving as a method of teaching may be used to accomplish the instructional goals of learning basic facts, concepts, and procedures, as well as goals for problem solving within problem contexts.

For example, if students investigate the areas of all triangles having a fixed perimeter of 60 units, the problem solving activities should provide ample practice in computational skills and use of formulas and procedures, as well as opportunities for the conceptual development of the relationships between area and perimeter.

Wilson, Fernandez, & Hadaway 1993

Activity Examples of Problem Solving As a Process for Learning

1. Review one or two sample lessons.
2. What is the mathematics students will learn through the lessons?
3. How do these lessons relate to the Standards for Mathematical Practice?
4. What is the role of the teacher while students are working through the lessons?
**CCSSM Practice 4 – Model with Mathematics**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

- In early grades, this might be as simple as writing an addition equation to describe a situation.
- In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
- By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

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**PARCC and SBAC – Three Foci**

- Concepts and procedures
- Communicate reasoning
- Problem solving, modeling, and data analysis

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**activity  Solve an Application Problem**

1. **Individual work:**
   - Solve an application problem of your choice.

2. **Pair work:**
   - Share and compare work with a partner.

3. **Group work:**
   - Discuss observations, issues, or challenges you see with the problems solved in your group.

4. **Whole class:** Discuss with the whole class.

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**Thank you!**

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