



Consortium on Reaching Excellence in Education®

Your Implementation Partner for Literacy and Math Achievement

Problem Solving in Elementary Math

Participant Handout

CORE and CORE Math are registered trademarks of Consortium on Reaching Excellence in Education, Inc.

Copyright © 2013 Consortium on Reaching Excellence in Education, Inc. All rights reserved. Printed in the United States of America. This publication is protected by copyright, and permission should be obtained from the publisher prior to any reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise.

Version 1.0

For information about or to gain permission to use any content in this document, write to:

Permissions, Editorial Department
Consortium on Reaching Excellence in Education, Inc.
2550 Ninth Street, Suite 102
Berkeley, California 94710
Phone: (888) 249-6155
Fax: (888) 460-4520
Email: info@corelearn.com
www.corelearn.com

CORE Mission

CORE serves as a trusted advisor at all levels of preK–12 education, working collaboratively with educators to support literacy and math achievement growth for all students.

Our implementation support services and products help our customers build their own capacity for effective instruction by laying a foundation of research-based knowledge, supporting the use of proven tools, and developing leadership.

As an organization committed to integrity, excellence, and service, we believe that with informed school and district administrators, expert teaching, and well-implemented programs, all students can become proficient academically.

**IES Practice Guide:
Improving Mathematical Problem Solving in Grades 4 Through 8**

1. Prepare problems and use them in whole-class instruction.

- Include both routine and nonroutine problems in problem-solving activities.
- Ensure that students will understand the problem by addressing issues students might encounter with the problem's context or language.
- Consider students' knowledge of mathematical content when planning lessons.

2. Assist students in monitoring and reflecting on the problem-solving process.

- Provide students with a list of prompts to help them monitor and reflect during the problem-solving process.
- Model how to monitor and reflect on the problem-solving process.
- Use student thinking about a problem to develop students' ability to monitor and reflect.

3. Teach students how to use visual representations.

- Select visual representations that are appropriate for students and the problems they are solving.
- Use think-alouds and discussions to teach students how to represent problems visually.
- Show students how to convert the visually represented information into mathematical notation.

4. Expose students to multiple problem-solving strategies.

- Provide instruction in multiple strategies.
- Provide opportunities for students to compare multiple strategies in worked examples.
- Ask students to generate and share multiple strategies for solving a problem.

5. Help students recognize and articulate mathematical concepts and notations.

- Describe relevant mathematical concepts and notation, and relate them to the problem-solving activity.
- Ask students to explain each step used to solve a problem in a worked example.
- Help students make sense of algebraic notation.

The practice guide is available for free from IES and can be downloaded as a PDF file from the IES website: <http://ies.ed.gov/ncee/wwc/practiceguide.aspx?sid=16>

Table 1. Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$

	Total Unknown	Addend Unknown	Both Addends Unknown
Put Together/ Take Apart	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	<p>("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? $2 + ? = 5, 5 - 2 = ?$</p> <p>("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie?</p>	<p>(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$</p> <p>(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have?</p>	<p>(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$</p> <p>(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have?</p>

Common Core State Standards for Mathematics. (2010). National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington, D.C., p. 88.

Table 2. Common Multiplication and Division Situations

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, & $18 \div 3 = ?$	$? \times 6 = 18$ & $18 \div 6 = ?$
Equal Groups	<p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p><i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p>	<p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p>	<p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p>
Arrays, Area	<p>There are 3 rows of apples with 6 apples in each row. How many apples are there?</p> <p><i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?</p>	<p>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</p>	<p>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</p>
Compare	<p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p><i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</p> <p><i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?</p> <p><i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>
General	$a \times b = ?$	$a \times ? = p$, & $p \div a = ?$	$? \times b = p$, & $p \div b = ?$

Common Core State Standards for Mathematics. (2010). National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington, DC, p. 89.

Problem Solving As a Process for Learning Math Concepts

Sample Lessons/Activities

Grades K–1 CCSSM K.CC.5, K.OA.4,5; 1.NBT.2a,b	Grade 1 CCSSM 1.OA.3
Understand How Numbers Relate to 5 and 10	Commutative Property of Addition
<p>Students visualize numbers by seeing them concretely on five and ten frames and determine how much less or how much more numbers are than 5 or 10.</p>  <ul style="list-style-type: none"> • Give each student a five or ten frame and some chips. Call out a number and have students place that number of chips on their five or ten frame. Ask students to describe what their number looks like. Some may put chips in order, and some may have spaces in between. It is only important that they correctly show the quantity. • Next, ask students to tell you how many more chips would be needed to fill the five or ten frame—to make a 5 or 10. In other words, how far is their number from 5 or 10? • Additionally with a five or ten frame, students can show numbers greater than 5 or 10. For example, on a five frame, a student can show 8 as a five and three more chips on the side. With a ten frame, a student can show 12 as a full ten and two more chips on the side. This builds understanding of numbers in relationship to these benchmark values. 	<p>Students realize the commutative property of addition and that it does not apply to subtraction.</p> <ul style="list-style-type: none"> • Use concrete objects in each case to explore and analyze. • Students compare pairs of numbers added together, such as $3 + 5$ and $5 + 3$, $2 + 4$ and $4 + 2$, etc., and conjecture about whether or not the sum is always the same, even if they switch the order of the addends. • Students compare switching the numbers in a subtraction problem, such as $5 - 3$ and $3 - 5$, to see if the difference changes or remains the same. • In a whole-class discussion, clarify and make clear the mathematics property.

Grade 4	CCSSM 4.NF.1	Grade 5	CCSSM 5.MD.5
Creating Equivalent Fractions		Volume of Rectangular Prisms (Boxes)	
<p>Students determine how equivalent fractions are related numerically and link this to prior knowledge of multiplying any number by 1.</p> <ul style="list-style-type: none"> • Students compare selected fractions that they know are equal based on use of a tool such as fraction strips. • Students write equations showing the equal fractions, such as $\frac{1}{3} = \frac{2}{6} \qquad \frac{1}{3} = \frac{3}{9}$ • After looking at several such pairs, students identify a pattern in the relationships between the numbers in equal fractions and generalize this into a rule: “If you multiply the numerator and denominator by the same number, you get an equivalent fraction.” • Through class discussion, this rule is made explicit to everyone and then linked to multiplying by 1. • (“If you multiply the numerator and denominator both by 2, that is the same as multiplying the fraction by 2/2, and we know 2/2 = 1. What do we know about multiplying something by 1?”) 		<p>Students determine a shortcut or formula for the volume of a rectangular box, as well as a general formula they can use later with all prisms and cylinders.</p> <ul style="list-style-type: none"> • Students work in pairs or small groups filling boxes with cubes. • They begin with smaller boxes that can be entirely filled with the cubes they have available. • Next, they try a box they can only partially fill, covering the base and maybe a couple layers above the base. They have to predict how many layers of the base it would take to fill the box to the top. • Finally, they work on a box that they cannot even completely cover the base with. They have to predict how many cubes would cover the base and use their technique from the previous box to predict how many layers of the base it would take to fill the box. • Whole-class processing of this work should lead to two important formulas: 1) general formula for volume of prisms: area of base \times height; 2) specific formula for volume of rectangular prisms, $L \times W \times H$. 	

K: "The Counting Jar," adapted from *Investigations in Number, Data, and Space* (Scott Foresman, 1998)

Directions for teacher: Provide jars with 15–20 objects of different sizes and shapes. Repeat this activity many times over the course of the year, varying the number and types of objects in the jar. As students become proficient with 15–20 objects, increase the number of objects to 30–35.

Directions for students:

1. Count the number of objects in the jar.
2. Make a drawing that represents this number of objects.
3. Create an equivalent number of objects (use any other objects you want).

CORE Grade 1

1. Henry has 11 pennies and Marlene has 24 pennies. How many more pennies does Marlene have than Henry?

2. Suppose Henry actually has 11 dimes and Marlene has 24 pennies. How can they split the money up equally so they both have the same amount? Show how you determined the answer. Use drawings and/or words.

The perimeter of the rectangular state park shown is 42 miles.



A ranger estimates that there are 9 deer in each square mile of the park.

If this estimate is correct, how many total deer are in the park? Explain your answer using numbers, symbols, and words.

Mr. Torres sold a total of 30 boxes of sports cards at his store on Monday. These boxes contained only baseball cards and football cards.

- Each box contained 25 sports cards.
- He earned \$3 for each sports card he sold.
- He earned a total of \$1,134 from the football cards he sold.

What amount of money did Mr. Torres earn from the baseball cards?

[†] Partnership for Assessment of Readiness for College and Careers (PARCC). Downloaded from http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html.

* Smarter Balanced Assessment Consortium (SBAC). Downloaded from <http://www.smarterbalanced.org/sample-items-and-performance-tasks/>.

Problem Solving in Elementary Math



1

Objectives

- Recognize problem solving as a tool for learning and applying math and as a goal of learning in itself.
- Analyze problem-solving activities including sample assessments from the Partnership for Assessment of Readiness for College and Careers (PARCC) and Smarter Balanced Assessment Consortium (SBAC).

2



activity Ordering Cards Problem

- Closely watch the card trick performed by the facilitator.
- Work individually or in pairs to figure out how to correctly arrange 10 cards, numbered 1–10, so that you can perform the card trick.
- What strategies did you use to figure it out?
- What does this have to do with problem solving?

3

Define Problem Solving

Problem solving involves reasoning and analysis, argument construction, and the development of innovative strategies. These abilities are used not only in advanced mathematics topics—such as algebra, geometry, and calculus—but also throughout the entire mathematics curriculum beginning in kindergarten.

Institute of Education Sciences (IES), *Improving Mathematical Problem Solving in Grades 4–8*, 2012

4

Recommendations: IES Guide on Improving Problem Solving

1. Prepare problems and use them in whole-class instruction.
2. Assist students in monitoring and reflecting on the problem-solving process.
3. Teach students how to use visual representations.
4. Expose students to multiple problem-solving strategies.
5. Help students recognize and articulate mathematical concepts and notations.

Institute of Education Sciences (IES), *Improving Mathematical Problem Solving in Grades 4–8, 2012*

5

Page 3

CORE MATH

Three Perspectives on Problem Solving

- **Problem solving as a goal:** Learn about how to problem solve.
- **Problem solving as a process:** Extend and learn math concepts through solving selected problems.
- **Problem solving as a tool for applications and modeling:** Apply math to real-world or word problems, and use mathematics to model the situations in these problems.

6

CORE MATH

Problem Solving As a Goal



7

CORE MATH

IES Sample of Steps for Problem Solving

1. Identify the givens and goals of the problem.
2. Identify the problem type.
3. Recall similar problems to help solve the current problem.
4. Use a visual to represent and solve the problem.
5. Solve the problem.
6. Check the solution.

Institute of Education Sciences (IES), *Improving Mathematical Problem Solving in Grades 4–8, 2012*

8

CORE MATH

Research on Problem Solving

Schoenfeld in his 1992 review of the literature concluded that attempts to teach students to use general problem-solving strategies (e.g., draw a picture, identify the givens and goals, consider a similar problem) generally had not been successful. He recommended that better results might be obtained by developing and teaching more specific problem-solving strategies (that link more clearly to classes of problems) . . .

Lesh & Zawojewski 2007

9

CORE MATH

Addition & Subtraction Problem Types

	Result Unknown	Change Unknown	Start Unknown
Add To	$2 + 3 = ?$	$2 + ? = 5$	$? + 3 = 5$
Take From	$5 - 2 = ?$	$5 - ? = 3$	$? - 2 = 3$

	Total Unknown	Addend Unknown	Both Addends Unknown
Put Together/ Take Apart	$3 + 2 = ?$	$3 + ? = 5, 5 - 3 = ?$	$5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	$2 + ? = 5, 5 - 2 = ?$	$2 + 3 = ?, 3 + 2 = ?$	$5 - 3 = ?, ? + 3 = 5$

CCSSM 2010

10

Page 4

CORE MATH

Multiplication & Division Problem Types

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
Equal Groups	$3 \times 6 = ?$	$3 \times ? = 18$ $18 \div 3 = ?$	$? \times 6 = 18$ $18 \div 6 = ?$
Arrays, Area	$3 \times 6 = ?$	$3 \times ? = 18$ $18 \div 3 = ?$	$? \times 6 = 18$ $18 \div 6 = ?$
Compare	$3 \times 6 = ?$	$3 \times ? = 18$ $18 \div 3 = ?$	$? \times 6 = 18$ $18 \div 6 = ?$
General	$a \times b = ?$	$a \times ? = p, p \div a = ?$	$? \times b = p, p \div b = ?$

CCSSM 2010

11

Page 5

CORE MATH



activity Solving Problems – B1

- **K–2:** Sheila took all the dollar bills out of her two pockets. The total was \$9. How much did she have in each pocket? Show how you know the answer is correct.
- **3–5:** Mickey has three times as much money as Jimmy. Altogether Mickey and Jimmy have \$36. How much money does Jimmy have? Explain/show how you got the answer.

12

Pages 6-7

CORE MATH

Problem Solving As a Process to Extend and Add to Learning of Math Concepts, Procedures, and Facts



13

CORE MATH

Problem Solving As a Process/Method

Problem solving as a method of teaching may be used to accomplish the instructional goals of learning basic facts, concepts, and procedures, as well as goals for problem solving within problem contexts.

For example, if students investigate the areas of all triangles having a fixed perimeter of 60 units, the problem solving activities should provide ample practice in computational skills and use of formulas and procedures, as well as opportunities for the conceptual development of the relationships between area and perimeter.

Wilson, Fernandez, & Hadaway 1993

14

CORE MATH



activity Examples of Problem Solving As a Process for Learning

1. Review one or two sample lessons.
2. What is the mathematics students will learn through the lessons?
3. How do these lessons relate to the Standards for Mathematical Practice?
4. What is the role of the teacher while students are working through the lessons?

15

Pages 8-10

CORE MATH

Problem Solving As a Tool for Applications and Modeling



16

CORE MATH

CCSSM Practice 4 – Model with Mathematics

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

- In early grades, this might be as simple as writing an addition equation to describe a situation.
- In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
- By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

17

CORE MATH

PARCC and SBAC – Three Foci

- Concepts and procedures
- Communicate reasoning
- Problem solving, modeling, and data analysis

18

CORE MATH



activity Solve an Application Problem

1. Individual work:

- Solve an application problem of your choice.

2. Pair work:

- Share and compare work with a partner.

3. Group work:

- Discuss observations, issues, or challenges you see with the problems solved in your group.

4. Whole class: Discuss with the whole class.

19

Pages 11-12

CORE MATH

CORE MATH

Thank you!
www.corelearn.com



20

CORE MATH

